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ROYAL ASTRONOMICAL SOCIETY.

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DR. J. W. L. GLAISHER, M.A., F.R.S., PRESIDENT, in the Chair;

Jnan Saran Chakravarti, M.A., Assistant Accountant-General, Allahabad, India;
Charles Sidney Mence, 49 Watling Street, London, E.C.;
Rev. John Stutter, O.S.B., Acton Burnell, near Shrewsbury; and
Ernest George Wainwright, B.A., St. John's College, Battersea, S.W.

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Spencer Lavington Fletcher, 38 Lammas Park Road, Ealing, W. (proposed by Rev. W. J. B. Roome); and
Professor Monroe B. Snyder, Director of the Philadelphia Observatory, Philadelphia, U.S.A. (proposed by Dr. Isaac Roberts).

The following were proposed by the Council as Associates of the Society:—

Professor W. W. Campbell, Director of the Lick Observatory, San José, California, U.S.A.;
Professor Julius Scheiner, Astrophysical Observatory, Potsdam, Germany; and
M. Charles Trépied, Director of the Observatory, Algiers.

Ninety-seven presents were announced as having been received since the last meeting, including, amongst others :—

A 9-inch Newtonian reflecting telescope and other astronomical instruments, formerly belonging to the late Canon Cross, F.R.A.S., presented by Mrs. Cross ; Die Triangulation von Java, Abtheilung VI., edited by J. A. C. Oudemans, presented in the name of the Government of the Netherlands by Professor Oudemans ; A. Laussedat, Recherches sur les instruments, les méthodes et le dessin topographiques, tome 2, presented by the author.

Formulæ and Tables for connecting Coordinates of Stars on Different Photographs, especially Different Plates of the Astrographic Chart. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. If (x_1y_1) are the rectilinear coordinates of a star on one plate, (x_2y_2) those of the same star on another, then generally

$$x_2=\frac{ax_1+by_1+c}{1+kx_1+ly_1}, \qquad y_2=\frac{dx_1+ey_1+f}{1+kx_1+ly_1} \qquad \ldots \quad (1)$$

where a, b, c, d, e, f, k, l are constants for the pair of plates. Of these eight constants k and l are known with sufficient accuracy as the approximate coordinates of one centre referred to the other, and the remaining six may be determined either (1) from the measures themselves directly, or (2) knowing (x_1y_1) in terms of standard coordinates for the first plate $(\xi_1\eta_1)$; also (x_2y_2) similarly in terms of $(\xi_2\eta_2)$ standard coordinates for the second plate ; and finally knowing $(\xi_2\eta_2)$ in terms of $(\xi_1\eta_1)$ from geometrical considerations, we can deduce the values of (x_2y_2) in terms of (x_1y_1).

It is important to be able to use both these methods. The second of them involves three numerical transformations which may be rather laborious ; and the present paper is an attempt to put these transformations into a simple form for the computer.

Mechanical Rule of Procedure.

2. A simple mechanical rule of combining two transformations such as

$$\begin{matrix} X=a_1x+a_2y \\ Y=b_1x+b_2y \end{matrix} \quad \text{with} \quad \begin{matrix} \xi=A_1X+A_2Y \\ \eta=B_1X+B_2Y \end{matrix}$$

has been found useful. The result is of course

$$\xi=(A_1a_1+A_2b_1)x+(A_1a_2+A_2b_2)y=\alpha_1x+\alpha_2y$$
$$\eta=(B_1a_1+B_2b_1)x+(B_1a_2+B_2b_2)y=\beta_1x+\beta_2y$$

and to form α and β by a mechanical process we notice that if we write down the coefficients in determinantal form as follows

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \text{ and } \begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix}$$

and then multiply the first *row* of the second by the *columns* of the first in turn we get the first *row* of the result, the second row being given by using the second row of the second determinant. If we rewrite the first determinant, substituting columns for rows, we then multiply rows by rows, which helps the eye appreciably.

3. In the work which follows we are often concerned with sets of coefficients which are conveniently written as follows :—

$$\begin{vmatrix} 1+a_1 & a_2 \\ b_1 & 1+b_2 \end{vmatrix} \text{ and } \begin{vmatrix} 1+A_1 & A_2 \\ B_1 & 1+B_2 \end{vmatrix}$$

all the letters denoting small quantities. In such cases the result takes a particular form which is worthy of note. If we omit unity in the terms of the leading diagonal we get the determinants of the last paragraph, and if we multiply these as before, and add to each term both of the corresponding terms of the factors, and finally replace unity in the leading diagonal, we get the result required for the expressions of the present paragraph.

Thus the first term is

$$1+(a_1+A_1)+(A_1a_1+A_2b_1)$$

where $(A_1a_1+A_2b_1)$ represents the term as it would be formed if the factors were as in § 2 ; a_1+A_1 is the sum of the first terms of the two factors ; and unity is added because the term occurs in the diagonal.

The rule is simpler in practice than in statement, as will be found on trial.

4. Both of the preceding paragraphs, though stated, for simplicity's sake, for determinants of four terms only, hold for any number of terms. As a numerical example suppose

$$\left.\begin{aligned} \xi_1 &= (1-\cdot00767)x_1 - \cdot00841y_1 + \cdot0833 \\ \eta_1 &= +\cdot00852x_1 + (1-\cdot00755)y_1 + \cdot2285 \end{aligned}\right\} \quad \dots \quad (2)$$

and also

$$\left.\begin{aligned} \xi_2 &= (1-\cdot00003)\xi_1 + \cdot00751\eta_1 - 10\cdot8766 \\ \eta_2 &= -\cdot00751\xi_1 + (1-\cdot00003)\eta_1 - 11\cdot9594 \end{aligned}\right\} \quad \dots \quad (3)$$

and we wish to find $(\xi_2\eta_2)$ in terms of (x_1y_1). We have only two variables in each case, but three terms in each equation. We complete each set by adding unity as a third variable, writing under the equations (2)

$$1 = 0 \,.\, x_1 + 0 \,.\, y_1 + 1 \times 1$$

so that the determinant for expressing $(\xi_1\eta_1 1)$ in terms of $(x_1y_1 1)$ is (omitting unity in the diagonal)

$$\begin{vmatrix} -\cdot00767 & -\cdot00841 & +\cdot0833 \\ +\cdot00852 & -\cdot00755 & +\cdot2285 \\ 0 & 0 & 0 \end{vmatrix} \quad \ldots \quad (4)$$

and that for expressing $(\xi_2\eta_2 1)$ in terms of $(\xi_1\eta_1 1)$ is

$$\begin{vmatrix} -\cdot00003 & +\cdot00751 & -10\cdot8766 \\ -\cdot00751 & -\cdot00003 & -11\cdot9594 \\ 0 & 0 & 0 \end{vmatrix} \quad \ldots \quad (5)$$

To find that for $(\xi_2\eta_2 1)$ in terms of $(x_1y_1 1)$ we first form the products of these as they stand, thus :

$$\begin{aligned} -\cdot00003\times(-\cdot00767)+\cdot00751\times(+\cdot00852)+0 &= +\cdot00006 \\ -\cdot00003\times(-\cdot00841)+\cdot00751\times(-\cdot00755)+0 &= -\cdot00006 \\ -\cdot00003\times(+\cdot0833)+\cdot00751\times(+\cdot2285)+0 &= +\cdot0016 \end{aligned}$$

and so on. The result is

$$\begin{vmatrix} +\cdot00006 & -\cdot00006 & +\cdot0016 \\ +\cdot00006 & +\cdot00006 & -\cdot0006 \\ 0 & 0 & 0 \end{vmatrix} \quad \ldots \quad (6)$$

Now add the corresponding terms in (4), (5), and (6), and replace unity in the diagonal. We get for our result

$$\begin{vmatrix} 1-\cdot00764 & -\cdot00096 & -10\cdot7917 \\ +\cdot00107 & 1-\cdot00752 & -11\cdot7315 \\ 0 & 0 & 1 \end{vmatrix} \quad \ldots \quad (7)$$

which means that

$$\left.\begin{aligned} \xi_2 &= (1-\cdot00764)x_1-\cdot00096y_1 \quad -10\cdot7917 \\ \eta_2 &= +\cdot00107x_1+(1-\cdot00752)y_1-11\cdot7315 \end{aligned}\right\} \quad \ldots \quad (8)$$

The arrangement of the work can be varied a little according to individual taste or according to the quantities involved.

5. The rules are equally convenient in algebraical transformations, of which some examples follow. Before giving them attention may be called to a particular case.

Let three direction-cosines $(l\ m\ n)$ be given in terms of $(\lambda\ \mu\ \nu)$ by the equations

$$\left.\begin{aligned} l &= a_1\lambda+a_2\mu+a_3\nu \\ m &= b_1\lambda+b_2\mu+b_3\nu \\ n &= c_1\lambda+c_2\mu+c_3\nu \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

and let three others (L, M, N) be given also in *terms of the same* $(\lambda\, \mu\, \nu)$ by equations with capital letters for the coefficients.

$$L = A_1\lambda + A_2\mu + A_3\nu \text{ \&c. } \ldots \quad \ldots \quad \ldots \quad (10)$$

and suppose we wish to find (L M N) in terms of $(l\, m\, n)$ by eliminating $(\lambda\, \mu\, \nu)$. This is not the same as the operation already considered; but we can make it the same by inverting equations (9) so as to get $(\lambda\, \mu\, \nu)$ in terms of $(l\, m\, n)$. Now by the properties of direction-cosines we do this by writing the coefficients $a_1\, b_1\, c_1$, which occur in the first *column* of (9), as the first *row* of the new equations; *i.e.*

$$\lambda = a_1 l + b_1 m + c_1 n \text{ \&c. } \ldots \quad \ldots \quad \ldots \quad (11)$$

and now, according to our rule, to get (L M N) in terms of $(l\, m\, n)$ we multiply the rows of $\begin{vmatrix} A_1 A_2 A_3 \\ B_1 B_2 B_3 \\ C_1 C_2 C_3 \end{vmatrix}$ by the columns of $\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix}$; or if we keep this latter in its original form (without changing rows for columns) we multiply *rows* by *rows*. Thus to eliminate $(\lambda\, \mu\, \nu)$ from two sets of equations like (9) and (10) *when the variables are direction-cosines* we multiply rows by rows as they stand; so that

$$L = a_1 l + a_2 m + a_3 n \text{ \&c.}$$

where
$$a_1 = A_1 a_1 + A_2 a_2 + A_3 a_3 \text{ \&c.}$$
$$a_2 = A_1 b_1 + A_2 b_2 + A_3 b_3$$

6. We proceed to consider the relations between standard coordinates on two different plates. The problem has been treated before (see *Monthly Notices*, liv. pp. 20 and 573), but the method indicated in the preceding paragraphs allows us to arrive at practical formulæ much more easily. We begin with direction-cosines as suggested in a former paper (*Monthly Notices*, lx. p. 201).

Let L M N represent the particular system

$$L = \sin\alpha\cos\delta \quad M = -\cos\alpha\cos\delta \quad N = \sin\delta$$

so that standard coordinates (ξ, η) on a plate with its centre at the pole are

$$\xi = L/N \qquad \eta = M/N$$

assuming that ξ and η are expressed in *circular measure*.

7. We now find (l, m, n) the coordinates referred to a plate-centre (A, D) in two steps.

First rotate the axis through an angle A round the axis of N so as to get

$$\left.\begin{aligned} X &= L\cos A + M\sin A \\ Y &= -L\sin A + M\cos A \\ Z &= N \end{aligned}\right\} \quad \ldots \quad \ldots \quad (12)$$

and next rotate round the axis of X through an angle $90°-D$, getting

$$\left.\begin{aligned} l &= X \\ m &= Y\sin D + Z\cos D \\ n &= -Y\cos D + Z\sin D \end{aligned}\right\} \quad \ldots \quad \ldots \quad (13)$$

Eliminating (X Y Z) in the manner indicated, the determinant for expressing (l, m, n) in terms of (L M N) is

$$\begin{vmatrix} \cos A & \sin A & 0 \\ -\sin A\sin D & \cos A\sin D & \cos D \\ \sin A\cos D & -\cos A\cos D & \sin D \end{vmatrix} \quad \ldots \quad (14)$$

For clearness it may be remarked that $(l\ m\ n)$ are such that the standard coordinates referred to a plate-centre whose R.A. is A and declination D are

$$\xi = l/n \qquad \eta = m/n$$

8. Now let $(l_1\ m_1\ n_1)$ refer to a plate-centre (A, D_1) and $(l_2\ m_2\ n_2)$ to a plate-centre (A, D_2) with the same R.A., but differing in declination. Then $(l_1\ m_1\ n_1)$ and $(l_2\ m_2\ n_2)$ are both given in terms of (L M N) by determinants like (14), writing D_1 and D_2 respectively in place of D. To eliminate (L M N) and find $(l_2\ m_2\ n_2)$ in terms of $(l_1\ m_1\ n_1)$ we use the rule of § 5, multiplying rows by rows as they stand. The result is

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & +\cos(D_2-D_1) & -\sin(D_2-D_1) \\ 0 & +\sin(D_2-D_1) & +\cos(D_2-D_1) \end{vmatrix} \quad \ldots \quad (15)$$

which is independent of A and D and depends on the difference (D_2-D_1) simply. For the plates of the Astrographic Chart this is always either 1° or 2°, the overlap in the second case being small. If we put $\tau = \tan(D_2-D_1)$, then

$$\sec(D_2-D_1) = (1+\tau^2)^{\frac{1}{2}} = 1+\tfrac{1}{2}\tau^2-\tfrac{1}{8}\tau^4+ \text{ \&c.}$$

If $\tau = \tan 1° = \cdot 017455$, $\frac{1}{8}\tau^4 = \cdot 00000001 = 0''\cdot 002$ and may be neglected. If $\tau = \tan 2°$, $\frac{1}{8}\tau^4 = 0''\cdot 03$ and can usually be neglected. For the present we shall assume that fourth powers can be neglected.

9. Since ξ and η are only concerned with the *ratios* of l and m to n, we may divide out by any common factor, such as $\cos(D_2-D_1)$, and thus (15) takes the form

$$\begin{vmatrix} 1+\frac{1}{2}\tau^2 & 0 & 0 \\ 0 & 1 & -\tau \\ 0 & \tau & 1 \end{vmatrix} \quad \ldots \quad \ldots \quad (16)$$

10. Next let us change the R.A. of the plate-centre without changing the declination; *i.e.* we have to multiply two determinants such as (14), in which first A_1 and secondly A_2 is substituted for A. The result is, writing α for A_2-A_1

$$\begin{vmatrix} \cos\alpha & \sin D\sin\alpha & -\cos D\sin\alpha \\ -\sin D\sin\alpha & \cos^2 D+\sin^2 D\cos\alpha & \sin D\cos D(1-\cos\alpha) \\ \cos D\sin\alpha & \sin D\cos D(1-\cos\alpha) & \sin^2 D+\cos^2 D\cos\alpha \end{vmatrix} \quad (17)$$

11. Put $\sin\alpha\cos D=\gamma$. Then γ is always small, for when D becomes large, A_2-A, or α increases approximately as sec D. For the plates of the astrographic chart γ is kept as nearly as possible at 1° (or 2°) and is of the same order as τ in the preceding paragraph. But if we write $t=\tan D$, then $\sin(A_2-A_1)\sin D=\gamma t$, and may become large.

Now,

$$1-\cos\alpha=2\sin^2\frac{\alpha}{2}=\tfrac{1}{2}\gamma^2\sec^2 D\sec^2\frac{\alpha}{2}$$
$$=\tfrac{1}{2}\gamma^2(1+t^2)+\tfrac{1}{8}\gamma^4(1+t^2)^2+\ \&c.$$

The first term is sufficient except near the Pole. The value of the second term at various values of D is as follows, assuming $\gamma=\tan 1^\circ$:

D =	40°	45°	50°	55°	60°
Value =	0″·007	0″·009	0″·013	0″·020	0″·035

For the present we will neglect the second term; and we must remember that the following formulæ are not accurate beyond D=60°.

12. With this restriction (17) may be written:

$$\begin{vmatrix} 1-\frac{1}{2}\gamma^2(1+t^2) & +\gamma t & -\gamma \\ -\gamma t & 1-\frac{1}{2}\gamma^2 t^2 & +\frac{1}{2}\gamma^2 t \\ +\gamma & +\frac{1}{2}\gamma^2 t & 1-\frac{1}{2}\gamma^2 \end{vmatrix} \quad . \quad . \quad (18)$$

As before we may divide out by $1-\frac{1}{2}\gamma^2$; and it seems an obvious simplification to write a single symbol for $\gamma/(1-\frac{1}{2}\gamma^2)$. But experience shows that nothing is ultimately gained by this.

Indeed, we have already made a false step as regards ultimate simplicity. Instead of writing

$$\gamma = \sin(A_2 - A_1)\cos D$$

it will be found better to have

$$\beta = (A_2 - A_1)\cos D$$

so that β is directly proportional to the difference of R.A. of the centres. Making this substitution so that

$$\gamma = \beta - \tfrac{1}{6}\beta^3(t^2+1)$$

the determinant (18) becomes

$$\begin{vmatrix} 1-\frac{1}{2}\beta^2t^2 & \beta t & -\beta-\frac{1}{3}\beta^3+\frac{1}{6}\beta^3t^2 \\ -\beta t & 1-\frac{1}{2}\beta^2(t^2-1) & \frac{1}{2}\beta^2 t \\ \beta & 0 & 1 \end{vmatrix} . \quad (19)$$

in which, however, some terms have been neglected, viz. the last column which multiplies n (which is near unity) is correct to the third power of β; but in the first two which multiply the small quantities (l and m or ultimately ξ and η) cubes of β have been neglected; and in the last line of these two columns, which ultimately multiplies squares of ξ and η, only the first powers are retained.

13. Let us now combine the two operations. We may move the centre first in R.A. and then in declination, or *vice versâ*. In the first case β will denote $(A_2-A_1)\cos D_1$, and in the second case $(A_2-A_1)\cos D_2$. On trial the formulæ are a little simpler in the first case; and we therefore move the centre

First in R.A. as in (19)
Next in decl. as in (16)

so that β denotes $(A_2-A_1)\cos D_1$.

Combining (19) and (16) as before we find, neglecting *squares* of β and τ in the first two terms of third row, *cubes* in first two terms of first and second rows, and fourth powers in the last column:

$$\begin{vmatrix} 1+\frac{1}{2}(\tau^2-\beta^2t^2), & \beta t, & -\beta-\frac{1}{3}\beta^3+\frac{1}{6}\beta^3t^2-\frac{1}{2}\beta\tau^2 \\ -\beta(t+\tau), & 1-\frac{1}{2}\beta^2(t^2-1), & -\tau+\frac{1}{2}\beta^2 t \\ \beta & \tau & 1 \end{vmatrix} \quad (20)$$

The term at the right-hand bottom corner is really $1+\frac{1}{2}\beta^2\tau t$; but on dividing by this factor throughout no terms are introduced of the order to be retained.

This gives the relations between $(l_2\ m_2\ n_2)$ and $(l_1\ m_1\ n_1)$; or

if $\xi\eta$ denote standard coordinates *expressed in circular measure* so that

$$\xi = l/n \qquad \eta = m/n$$

or $$l = n\xi \qquad m = n\eta \qquad n = n \times 1$$

then since we are ultimately concerned with ratios only, we may omit the common factor n, and the above determinant gives the relations between $(\xi_2\ \eta_2\ 1)$ and $(\xi_1\ \eta_1\ 1)$.

14. But in practice we should not compare $(\xi_2\ \eta_2\ 1)$ with $(\xi_1\ \eta_1\ 1)$ as they stand. We should first add to one set the small terms representing the effect of the denominator. Writing the relations

$$\xi_2 = \frac{(1+a)\xi_1 + b\eta_1 + c}{1 + k\xi_1 + l\eta_1} \qquad \eta_2 = \frac{d\xi_1 + (1+e)_1 + f}{1 + k\xi_1 + l\eta_1} \quad \ldots \quad (21)$$

[where $a = \frac{1}{2}(\tau^2 - \beta^2 t^2)$, $b = \beta(t-\tau)$, &c., in (16)]

we can rewrite the first of these

$$\xi_2 + k\xi_1\xi_2 + l\eta_1\xi_2 = (1+a)\xi_1 + b\eta_1 + c \quad \ldots \quad \ldots \quad (22)$$

The terms added to ξ_2 on the left are small, and we may in them put with sufficient accuracy

$$\xi_2 = \xi_1 + c \qquad \eta_2 = \eta_1 + f$$

It is shown in a former paper (*Monthly Notices*, lv. p. 108) that the most convenient way to apply these small terms to $(\xi_2\eta_2)$ is to move the origin (for them only) to the point midway between the two plate-centres, *i.e.* the point

$$\xi_1 = -\tfrac{1}{2}c \qquad \eta_1 = -\tfrac{1}{2}f$$

Thus we put $k\xi_1(\xi_1 + c) + l\eta_1(\xi_1 + c)$ into the form

$$\left.\begin{aligned} &k(\xi_1 + \tfrac{1}{2}c)^2 + l(\eta_1 + \tfrac{1}{2}f)(\xi_1 + \tfrac{1}{2}c) \\ &-\tfrac{1}{4}kc^2 - \tfrac{1}{4}lcf - \tfrac{1}{2}lf\xi_1 + \tfrac{1}{2}lc\eta_1 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (23)$$

of which the second line, containing only linear terms, can be added to the right-hand side of equation (22). Or writing

$$\left.\begin{aligned} \mathrm{P} &= k(\xi_1 + \tfrac{1}{2}c)^2 + l(\eta_1 + \tfrac{1}{2}f)(\xi_1 + \tfrac{1}{2}c) \\ \mathrm{Q} &= k(\xi_1 + \tfrac{1}{2}c)(\eta_1 + \tfrac{1}{2}f) + l(\eta_1 + \tfrac{1}{2}f)^2 \end{aligned}\right\} \quad \ldots \quad (24)$$

we have

$$\left.\begin{aligned} \xi_2 + \mathrm{P} &= (1 + a + \tfrac{1}{2}lf)\xi_1 + (b - \tfrac{1}{2}lc)\eta_1 + c + \tfrac{1}{4}kc^2 + \tfrac{1}{4}lcf \\ \eta_2 + \mathrm{Q} &= (d - \tfrac{1}{2}kf)\xi_1 + (1 + e + \tfrac{1}{2}kc)\eta_1 + f + \tfrac{1}{4}lf^2 + \tfrac{1}{4}kcf \end{aligned}\right\} \quad (25)$$

In the paper above referred to it is shown that the corrections P and Q are sensibly zero over a large part of the region common to the two plates, and only reach $1''{\cdot}0$ in the corners of this region. They are easily applied, from approximate values of the coordinates, by means of a small table or a diagram.

15. Returning now to (20) and (24) we see that we may put

$$c=-\beta \qquad f=-\tau \qquad k=\beta \qquad l=\tau$$

and thus

$$\left.\begin{aligned} \mathrm{P} &= +\beta(\xi_1-\tfrac{1}{2}\beta)^2+\tau(\xi_1-\tfrac{1}{2}\beta)(\eta_1-\tfrac{1}{2}\tau) \\ \mathrm{Q} &= +\beta(\xi_1-\tfrac{1}{2}\beta)(\eta_1-\tfrac{1}{2}\tau)+\tau(\eta_1-\tfrac{1}{2}\tau)^2 \end{aligned}\right\} \quad \ldots \quad (26)$$

and substituting the values of a, b, c, &c., we find for $\xi_2+\mathrm{P}$, $\eta_2+\mathrm{Q}$ in terms of $(\xi_1\eta_1)$, the determinant (instead of (20))

$$\begin{vmatrix} 1-\frac{1}{2}\beta^2t^2, & \beta(t+\frac{1}{2}\tau), & -\beta-\frac{1}{12}\beta^3+\frac{1}{6}\beta^3t^2-\frac{1}{4}\beta\tau^2 \\ -\beta(t+\frac{1}{2}\tau), & 1-\frac{1}{2}\beta^2t^2, & -\tau+\frac{1}{2}\beta^2t+\frac{1}{4}\tau^3+\frac{1}{4}\beta^2\tau \\ 0 & 0 & 1 \end{vmatrix} \quad (27)$$

16. The expressions in the third column differ from β and τ by small quantities of which $-\frac{1}{2}\beta^2t$ alone is of the second order. Had we taken the circular measure of 1° for τ instead of the tangent, we should have had instead of $\frac{1}{4}\tau^3$

$$\tfrac{1}{4}\tau^3-\tfrac{1}{3}\tau^3=-\tfrac{1}{12}\tau^3$$

as in the case of β. These expressions lend themselves readily to tabulation for different values of β and τ; for if we calculate the values of (say)

$$-\beta, \quad -\tfrac{1}{4}\beta\tau^2, \text{ and } +\tfrac{1}{6}\beta^3(t^2-\tfrac{1}{2})$$

for $\beta=1^{\text{min}}$, $\tau=1°$, then to get the values for $\beta=m^{\text{min}}$, $\tau=n°$, we multiply these terms by m, mn, and m^3 respectively.

17. But the most important feature of the determinant (27) is the form of the four terms in the top left corner. If we put

$$\sin\theta=\beta(t+\tfrac{1}{2}\tau)$$

then to the order of approximation named we have

$$\cos\theta=1-\tfrac{1}{2}\beta^2t^2$$

so that the determinant may be written

$$\begin{vmatrix} \cos\theta & \sin\theta & -\mathrm{C} \\ -\sin\theta & \cos\theta & -\mathrm{F} \\ 0 & 0 & 1 \end{vmatrix} \quad \ldots \quad \ldots \quad (28)$$

or otherwise

$$\left.\begin{aligned} \xi_2+\mathrm{P} &= \xi_1\cos\theta+\eta_1\sin\theta-\mathrm{C} \\ \eta_2+\mathrm{Q} &= -\xi_1\sin\theta+\eta_1\cos\theta-\mathrm{F} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (29)$$

so that $\xi_2+\mathrm{P}$, $\eta_2+\mathrm{Q}$ can be obtained from $(\xi_1\eta_1)$ by rotation of the axes through an angle θ and a change of origin. This is not only interesting theoretically but is an important practical gain, since we have only two quantities ($\sin\theta$ and $\cos\theta$) to tabulate instead of four, as in (20). [We might indeed say that we

have only one, viz. the argument θ; but it will be more convenient for the computer to have sin θ and cos θ given him.] This proposition has only been proved in an approximate form at present; but it is accurately and generally true, as will now be shown by an independent proof.

18. Suppose we have in the general case

$$X = \frac{a_1x+a_2y+a_3}{c_1x+c_2y+c_3} \qquad Y = \frac{b_1x+b_2y+b_3}{c_1x+c_2y+c_3} \quad \ldots \quad (30)$$

where $a_1a_2a_3$, $b_1b_2b_3$, $c_1c_2c_3$ are a set of direction cosines with the well known properties

$$a_1 = b_2c_3 - b_3c_2, \qquad b_1 = a_3c_2 - a_2c_3 \text{ \&c.}$$
$$a_1^2+a_2^2+a_3^2 = 1, \qquad a_1a_2+b_1b_2+c_1c_2 = 0 \text{ \&c.}$$

Then
$$c_3X + X(c_1x+c_2y) = a_1x+a_2y+a_3 \quad \ldots \quad \ldots \quad (31)$$

In the products $X(c_1x+c_2y)$ let us move the origins of coordinates through distances to be presently determined.

Thus write

$$X(c_1x+c_2y) = (X-k)\{c_1(x-\alpha)+c_2(y-\beta)\}$$
$$+X(c_1\alpha+c_2\beta)+k\{c_1x+c_2y-c_1\alpha-c_2\beta\} \quad \ldots \quad (32)$$

Denoting $(X-k)\{c_1(x-\alpha)+c_2(y-\beta)\}$ by P we have on combining (31) and (32)

$$(c_3+c_1\alpha+c_2\beta)X + P = (a_1-kc_1)x+(a_2-kc_2)y+a_3$$
$$+kc_1\alpha+kc_2\beta \quad \ldots \quad (33)$$

Similarly

$$(c_3+c_1\alpha+c_2\beta)Y + Q = (b_1-lc_1)x+(b_2-lc_2)y+b_3+lc_1\alpha+lc_2\beta \quad (34)$$

We have in these equations moved the origin of (X, Y) in the small terms P and Q to the point (k, l) and the origin of (x, y) to the point (α, β).

19. Now if the transformations on the right of (33) and (34) represent a rotation (with possible change of scale also) we have

$$\left.\begin{aligned} a_1-kc_1 &= b_2-lc_2 \\ \text{and} \qquad a_2-kc_2+b_1-lc_1 &= 0 \end{aligned}\right\} \ldots \quad \ldots \quad \ldots \quad (35)$$

These equations give

$$k = \frac{a_1c_1-b_2c_1+a_2c_2+b_1c_2}{c_1^2+c_2^2} = \frac{-a_3c_3+a_3}{1-c_3^2}$$

$$\left.\begin{aligned} \text{or} \qquad k &= \frac{a_3}{1+c_3} \\ \text{and} \qquad l &= \frac{b_3}{1+c_3} \quad \text{similarly} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (36)$$

The amount of rotation θ is given by

$$\sin\theta = a_2 - kc_2 = -(b_1 - lc_1) = \frac{a_2 - b_1}{1 + c_3} \quad \ldots \quad \ldots \quad (37)$$

and the rotation will be unaccompanied by any change of scale if

$$\cos\theta = a_1 - kc_1 = b_2 - lc_2 = \frac{a_1 + b_2}{1 + c_3} \quad \ldots \quad \ldots \quad (38)$$

i.e. if $(a_2 - b_1)^2 + (a_1 + b_2)^2 = (1 + c_3)^2$

or $a_1^2 + a_2^2 + b_1^2 + b_2^2 + 2(a_1 b_2 - a_2 b_1 - c_3) = 1 + c_3^2$

which is readily seen to be true. To complete the conditions that X + P, Y + Q shall be deducible from x and y by a simple rotation and change of origin, we must choose α and β so that the common coefficient of X and Y in equations (33) and (34) is unity, *i.e.* we must have

$$c_3 + c_1\alpha + c_2\beta = 1 \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

which gives a straight line on which (α, β) may lie.

If we put $\frac{\alpha}{c_1} = \frac{\beta}{c_2} = \gamma$ then $\gamma = \frac{1}{1 + c_3}$ and we have

$$\left.\begin{aligned} X + P &= x\cos\theta + y\sin\theta + \frac{2a_3}{1 + c_3} \\ Y + Q &= -x\sin\theta + y\cos\theta + \frac{2b_3}{1 + c_3} \end{aligned}\right\} \quad \ldots \quad (40)$$

20. These equations may be written in a variety of forms. In (40) the constant terms may be taken over to the left and subtracted from (X, Y), which is equivalent to moving the origin for (X, Y) to the point corresponding to the other plate-centre. If instead of this we move the origin of (x, y) to the plate-centre for (X Y) we may write the equations

$$\left.\begin{aligned} X + P &= \left(x - \frac{2c_1}{1 + c_3}\right)\cos\theta + \left(y - \frac{2c_2}{1 + c_3}\right)\sin\theta \\ Y + Q &= -\left(x - \frac{2c_1}{1 + c_3}\right)\sin\theta + \left(y - \frac{2c_2}{1 + c_3}\right)\cos\theta \end{aligned}\right\} \quad \ldots (41)$$

Or, finally, if we move *each origin halfway*, as has already been done in the case of the small terms P and Q, *i.e.* if we write

$$X_0 = X - \frac{a_3}{1 + c_3},\ Y_0 = Y - \frac{b_3}{1 + c_3},\ x_0 = x - \frac{c_1}{1 + c_3},\ y_0 = y - \frac{c_2}{1 + c_3} \quad (42)$$

then we have

$$\left.\begin{aligned} X_0 + P &= x_0\cos\theta + y_0\sin\theta \\ Y_0 + Q &= -x_0\sin\theta + y_0\cos\theta \end{aligned}\right\} \quad \ldots \quad (43)$$

where $P = X_0(c_1 x_0 + c_2 y_0) \qquad Q = Y_0(c_1 x_0 + c_2 y_0)$

This is theoretically the simplest form. Practically it is not so convenient, as it involves two additions or subtractions in place of one. We therefore use (40) or (41) in practice, in spite of the greater apparent simplicity of (43).

21. All this is quite rigorous and applies to any plates, however large. It may be noticed that $x_o y_o$ $X_o Y_o$ vanish together; and in the neighbourhood of this point, which is midway between the plate-centres, P and Q depend on the squares of small quantities at least, and are therefore small. If the plate-centres are as near together as those for the Astrographic Chart, P and Q never become large for the region common to the plates, and are readily applied by a small table or diagram. If, however, the plates are large, P and Q may be the most important parts of the formulæ and must be carefully calculated. We shall proceed no further with the consideration of such cases at present, but return to that where P and Q are small and readily applied, and especially to the case of plates of the Astrographic Chart.

22. In § 15 the determinant (27) gives the relations between two sets of standard coordinates in an approximate form, sufficiently accurate only when β and τ are small. But we can now form a more general expression. If $(A_1 D_1)$, $(A_2 D_2)$ be the plate-centres, and if $\alpha = A_2 - A_1$, then it can be easily found by procedure already indicated that the general determinant for transforming $(l_2 m_2 n_2)$ into $(l_1 m_1 n_1)$, or between $(\xi_2 \eta_2 1)$ and $(\xi_1 \eta_1 1)$ is

$$\left|\begin{array}{ll} \cos\alpha, & \sin D_1 \sin\alpha, \qquad -\cos D_1 \sin\alpha \\ -\sin D_2 \sin\alpha, & \cos D_1 \cos D_2 + \cos\alpha \sin D_1 \sin D_2, \\ & \qquad \sin D_1 \cos D_2 - \cos\alpha \cos D_1 \sin D_2 \\ \cos D_2 \sin\alpha, & \cos D_1 \sin D_2 - \cos\alpha \cos D_2 \sin D_1, \\ & \qquad \sin D_1 \sin D_2 + \cos\alpha \cos D_1 \cos D_2 \end{array}\right| \quad (44)$$

This is the relation *before* the quantities P and Q have been applied to one set; corresponding to the

$$\left|\begin{array}{lll} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{array}\right|$$

of equation (30). *After* the application of P and Q we have shown that we may write the determinant

$$\left|\begin{array}{lll} \cos\theta & \sin\theta & -C \\ -\sin\theta & \cos\theta & -F \\ 0 & 0 & 1 \end{array}\right| \quad \ldots \quad \ldots \quad (45)$$

where $\sin\theta = (a_2 - b_1)/(1 + c_3)$.

Thus $$a_2-b_1=\sin\alpha\,(\sin D_1+\sin D_2)$$
$$=2\sin\alpha\sin\tfrac{1}{2}(D_1+D_2)\cos\tfrac{1}{2}(D_2-D_1) \quad (46)$$
and $$1+c_3=1+\cos(D_2-D_1)-2\cos D_1\cos D_2\sin^2\tfrac{1}{2}\alpha$$
or $$\tfrac{1}{2}(1+c_3)=[1-\sin^2\tfrac{1}{2}(D_2-D_1)-\cos D_1\cos D_2\sin^2\tfrac{1}{2}\alpha] \quad (47)$$

Thus for Astrographic Chart plates, since even when α is large, cos D sin α is always of the order β or τ (about one degree), $\frac{1}{2}(1+c_3)$ differs from 1 by quantities of the second order. It can be quickly tabulated as far as the Pole.

23. The quantities C and F of (45) are given by

$$\left.\begin{aligned}\tfrac{1}{2}(1+c_3)C&=\cos D_1\sin\alpha\\ \tfrac{1}{2}(1+c_3)F&=\sin(D_2-D_1)-2\sin D_2\cos D_1\sin^2\tfrac{1}{2}\alpha\end{aligned}\right\}\ldots \quad (48)$$

and can also be quickly tabulated when α, D_2, and D_1 are given.

24. We have thus thrown the determinant for transforming one set of standard coordinates into another into a simple form (45), which lends itself to tabulation. The values of the quantities for the series of Astrographic Chart plates are tabulated at the end of this paper. This completes the theoretical part of the work. It remains to show by an example what numerical work is involved, and this we proceed to do.

Example of the Comparison of Plates.

25. Two plates were taken at the University Observatory, Oxford, with centres at (approximately)

$$A_1=1^h\,56^m \qquad D_1=+25^\circ \qquad A_2=2^h\,0^m \qquad D_2=+26^\circ$$

If (x_1y_1) denote measures on the first plate, referred to the *centre* of the *réseau* (if referred to the *corner* the transformation to centre is easily made), and $(\xi_1\eta_1)$ the corresponding standard coordinates, it was found from comparison of the measures for stars with their standard coordinates computed from meridian observations (Cambridge and Berlin) that

$$\left.\begin{aligned}\xi_1-x_1&=-\cdot00767x_1-\cdot00841y_1+\cdot0833\\ \eta_1-y_1&=+\cdot00852x_1-\cdot00755y_1+\cdot2285\end{aligned}\right\}\quad\ldots \quad (49)$$

the coordinates and constant terms being expressed in *réseau* intervals. The determinant for expressing $(\xi_1\eta_1 1)$ in terms of $(x_1y_1 1)$ is thus (omitting the unities in the diagonal)

$$\begin{vmatrix} -\cdot00767 & -\cdot00841 & +\cdot0833\\ +\cdot00852 & -\cdot00755 & +\cdot2285\\ 0 & 0 & 0\end{vmatrix} \quad\ldots \quad (50)$$

For expressing $(\xi_2\eta_2 1)$ in terms of $(\xi_1\eta_1 1)$, we refer to the tables at the end of this paper for $D_1 = 25°$, and find accordingly

$$\left|\begin{array}{ccc} -\cdot00003 & +\cdot00751 & -10\cdot8766 \\ -\cdot00751 & -\cdot00003 & -11\cdot9594 \\ 0 & 0 & 0 \end{array}\right| \quad \ldots \quad (51)$$

It will be seen on reference to § 4 that (50) and (51) are identical with (4) and (5); and the combination has already been performed as an example of the process. The result is

$$\left|\begin{array}{ccc} -\cdot00764 & -\cdot00096 & -10\cdot7917 \\ +\cdot00107 & -\cdot00752 & -11\cdot7315 \\ 0 & 0 & 0 \end{array}\right| \quad \ldots \quad (52)$$

which expresses $(\xi_2\eta_2 1)$ in terms of $(x_1y_1 1)$. We now want $(x_2y_2 1)$ in terms of $(\xi_2\eta_2 1)$. This is not given directly in the ordinary course. We ordinarily find $(\xi_2\eta_2 1)$ in terms of $(x_2y_2 1)$, as in the case of the first plate, viz. (referring to *centre* of *réseau*)

$$\left.\begin{array}{l} \xi_2 - x_2 = -\cdot00754x_2 - \cdot00200y_2 + \cdot1217 \\ \eta_2 - y_2 = +\cdot00209x_2 - \cdot00763y_2 + \cdot2284 \end{array}\right\} \quad \ldots \quad (53)$$

and these are to be inverted. Now instead of inverting them as they stand, *take the constant terms over to the left* and write

$$X = \xi_2 - \cdot1217 \qquad Y = \eta_2 - \cdot2284$$

It is obvious that X and Y are very easily substituted for ξ_2 and η_2 in equations (52). We merely subtract ·1217 and ·2284 from the constants in the last column; and the equations

$$\left.\begin{array}{l} X = (1 - \cdot00754)x_2 - \cdot00200y_2 \\ Y = +\cdot00209x_2 + (1 - \cdot00763)y_2 \end{array}\right\} \quad \ldots \quad (54)$$

are readily inverted into

$$\left.\begin{array}{l} \Delta \times x_2 = (1 - \cdot00763)X + \cdot00200Y \\ \Delta \times y_2 = -\cdot00209\,X + (1 - \cdot00754)Y \end{array}\right\} \quad \ldots \quad (55)$$

the coefficients on the right being written down by inspection of (54), and the value of Δ being

$$\begin{aligned} \Delta &= (1 - \cdot00763)(1 - \cdot00754) + \cdot00200 \times \cdot00209 \\ &= 1 - \cdot01511 = (1\cdot01534)^{-1} \end{aligned}$$

Multiplying the coefficients of (55) by Δ^{-1} we get

$$\left|\begin{array}{ccc} +\cdot00760 & +\cdot00203 & 0 \\ -\cdot00212 & +\cdot00769 & 0 \\ 0 & 0 & 0 \end{array}\right| \quad \ldots \quad \ldots \quad (56)$$

which is to be combined with (52) modified, viz. :

$$\left|\begin{array}{ccc} -\cdot00764 & -\cdot00096 & -10\cdot9134 \\ +\cdot00107 & -\cdot00752 & -11\cdot9599 \\ 0 & 0 & 0 \end{array}\right| \quad \ldots \quad (57)$$

Combining (56) and (57) we get finally

$$\left|\begin{array}{ccc} -\cdot00010 & +\cdot00104 & -11\cdot0207 \\ -\cdot00103 & +\cdot00010 & -12\cdot0288 \\ 0 & 0 & 0 \end{array}\right| \quad \ldots \quad (58)$$

or in the usual notation

$$\left.\begin{array}{l} x_2-x_1=-\cdot00010x_1+\cdot00104y_1-11\cdot0207 \\ y_2-y_1=-\cdot00103x_1+\cdot00010y_1-12\cdot0288 \end{array}\right\} \quad \ldots \quad (59)$$

in which we must remember that (x_2y_2) are supposed corrected by the small quantities P Q. From a *direct* comparison of the measured coordinates of 82 stars common to the plates we find (after similar correction by P Q)

$$\left.\begin{array}{l} x_2-x_1=-\cdot00012x_1+\cdot00088y_1-11\cdot0222 \\ y_2-y_1=-\cdot00090x_1-\cdot00011y_1-12\cdot0275 \end{array}\right\} \quad \ldots \quad (60)$$

The differences between the two formulæ are

$$\left.\begin{array}{ll} \text{in } x & +\cdot00002x_1+\cdot00016y_1-0\cdot0015 \\ \text{in } y & -\cdot00013x_1+\cdot00021y_1-0\cdot0013 \end{array}\right\} \quad \ldots \quad (61)$$

which are applicable over a square region common to the plates from about

$$x_1=-1 \text{ to } x=+12 \quad \text{and} \quad y_1=-0\cdot5 \text{ to } y=+12\cdot5.$$

The extreme values of the differences at the corners of this region are

Coords.	$-1, -\frac{1}{2}$	$-1, 12\frac{1}{2}$	$12, -\frac{1}{2}$	$12, 12\frac{1}{2}$
Error in x	$-0''\cdot48$	$+0''\cdot15$	$-0''\cdot40$	$+0''\cdot21$
Error in y	$-0\cdot38$	$+0\cdot43$	$-0\cdot88$	$-0\cdot06$

These quantities are made up partly of accidental errors and partly of the systematic errors of the catalogues ; and it is for the detection and discussion of such systematic differences that this double method of comparison of plates is useful.

Tables for Astrographic Chart Plates.

The coefficients in the equations

$$\xi_2 + P = \xi_1 \cos\theta + \eta_1 \sin\theta - C$$
$$\eta_2 + Q = \eta_1 \cos\theta - \xi_1 \sin\theta - F$$

are given in the following tables :—

Table I. (*Plates one above another.*)

$A_2 - A_1 = 0$ $\quad D_2 - D_1 = 2°$

$\sin\theta = (1 - \cos\theta) = 0$ $\quad C = 0$ $\quad F = 24{\cdot}0025$ (*All zones.*)

Table II. (*Plates side by side.*)

$D_2 - D_1 = 0$ $\quad A_2 - A_1$ ($= 2\alpha$ say) variable with D_1.

From $D = 0°$ to $D = 27°$: $2\alpha = 8^m$.

D	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	F.	Circular Measure. C.	F.
0°	·000000	·000000	24·0025	−0·0000	·0349102	−·0000000
1	000	·000509	23·9988	0·0073	·0349048	0106
2	001	·001218	23·9878	0·0146	·0348889	0213
3	002	·001827	23·9695	0·0219	·0348622	0318
4	003	·002435	23·9439	0·0291	·0348250	0424
5	005	·003042	23·9110	0·0363	·0347772	0529
6	·000007	·003649	23·8709	−0·0435	·0347188	−·0000633
7	009	·004254	23·8234	0·0506	·0346498	0737
8	012	·004858	23·7686	0·0577	·0345701	0840
9	015	·005461	23·7066	0·0646	·0344800	0941
10	018	·006062	23·6375	0·0716	·0343794	1042
11	·000022	·006661	23·5611	−0·0785	·0342683	−·0001141
12	026	·007258	23·4775	0·0852	·0341467	1239
13	032	·007853	23·3869	0·0918	·0340148	1335
14	036	·008445	23·2890	0·0983	·0338725	1430
15	041	·009035	23·1840	0·1047	·0337198	1523
16	·000046	·009622	23·0720	−0·1110	·0335569	−·0001614
17	052	·010206	22·9530	0·1171	·0333838	1703
18	058	·010788	22·8270	0·1231	·0332005	1790
19	065	·011365	22·6941	0·1289	·0330072	1875
20	071	·011940	22·5541	0·1346	·0328036	1958

D	$(1-\cos\theta)$	$\sin\theta$	Réseau Interva's. C.	F.	Circular Measure. C.	F.
21°	·000078	·012510	22·4072	−0·1401	·0325900	−·0002038
22	084	·013077	22·2536	0·1455	·0323666	2116
23	092	·013640	22·0933	0·1506	·0321334	2191
24	101	·014198	21·9262	0·1556	·0318903	2263
25	108	·014753	21·7524	0·1604	·0316376	2333
26	·000117	·015303	21·5720	−0·1650	·0313752	−·0002400
27	125	·015848	21·3850	0·1694	·0311032	2464
		From D = 28° to D = 36°: $2\alpha = 9^m$.				
28	·000170	·018437	23·8406	−0·2197	·0346748	−·0003196
29	181	·019039	23·6157	0·2247	·0343476	3269
30	193	·019636	23·3834	0·2295	·0340098	3338
31	205	·020226	23·1441	0·2340	·0336617	3404
32	217	·020810	22·8977	0·2382	·0333034	3465
33	·000229	·021388	22·6444	−0·2422	·0329350	−·0003522
34	241	·021960	22·3842	0·2457	·0325565	3574
35	254	·022524	22·1172	0·2490	·0321681	3622
36	266	·023082	21·8433	0·2520	·0317698	3666
		From D = 37° to D = 48°: $2\alpha = 10^m$.				
37	·000345	·026259	23·9587	−0·3145	·0348466	−·0004575
38	361	·026863	23·6398	0·3176	·0343827	4619
39	377	·027459	23·3137	0·3201	·0339084	4656
40	393	·028046	22·9804	0·3223	·0334237	4688
41	410	·028625	22·6402	0·3241	·0329289	4714
42	·000426	·029195	22·2931	−0·3255	·0324240	−·0004734
43	443	·029756	21·9393	0·3264	·0319094	4748
44	459	·030308	21·5787	0·3271	·0313850	4757
45	476	·030851	21·2115	0·3273	·0308509	4760
46	492	·031384	20·8379	0·3271	·0303075	4757
47	·000509	·031908	20·4580	−0·3264	·0297550	−·0004748
48	526	·032422	20·0718	0·3255	·0291933	4734

From D = 49° to D = 59°: $2\alpha = 12^m$.

D	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	F.	Circular Measure. C.	F.
49°	·000781	·039510	23·6143	−0·4667	·0343456	−·0006788
50	804	·040103	23·1363	0·4641	·0336504	6750
51	827	·040684	22·6513	0·4609	·0329450	6704
52	851	·041252	22·1594	0·4572	·0322296	6650
53	874	·041808	21·6609	0·4530	·0315045	6589
54	·000897	·042352	21·1556	−0·4482	·0307696	−·0006519
55	919	·042882	20·6439	0·4428	·0300254	6441
56	942	·043398	20·1261	0·4368	·0292722	6354
57	963	·043902	19·6020	0·4304	·0285100	6261
58	985	·044392	19·0721	0·4235	·0277392	6160
59	·001007	·044869	18·5363	0·4160	·0269599	6051

From D = 60° to D = 63°: $2\alpha = 16^m$.

60	·001829	·060429	23·9878	−0·7254	·0348888	−·0010551
61	1865	·061028	23·2586	0·7104	·0338283	10332
62	1900	·061608	22·5224	0·6945	·0327575	10101
63	1935	·062169	21·7793	0·6776	·0316767	09856

From D = 64° to D = 66°: $2\alpha = 18^m$.

64	·002490	·070539	23·6547	−0·8354	·0344044	−·0012150
65	2526	·071128	22·8042	0·8120	·0331674	11810
66	2574	·071694	21·9468	0·7877	·0319204	11457

From D = 67° to D = 70°: $2\alpha = 20^m$.

67	·003225	·080250	23·4209	−0·9413	·0340643	−·0013691
68	3272	·080832	22·4539	0·9090	·0326578	13221
69	3317	·081387	21·4801	0·8755	·0312416	12734
70	3361	·081918	20·4997	0·8411	·0298156	12233

From D = 71° to D = 74°: $2\alpha = 24^m$.

71	·004898	·098862	23·4049	−1·1598	·0340410	−·0016868
72	4956	·099438	22·2144	1·1072	·0323095	16104
73	5012	·099984	21·0172	1·0533	·0305683	15320
74	5063	·100500	19·8137	0·9982	·0288179	14518

From D = 75° to D = 78°: $2\alpha = 30^m$.

75	·007988	·126115	23·2339	−1·4709	·0337924	−·0021394
76	8056	·126681	21·7164	1·3811	·0315852	20087
77	8123	·127208	20·1922	1·2895	·0293684	18755
78	8187	·127697	18·6621	1·1964	·0271429	17401

From D = 79° to D = 82° : $2\alpha = 40^m$.

D	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	F.	Circular Measure. C.	F.
79°	·014647	·170505	22·7873	−1·9571	·0331428	−·0028464
80	14738	·171049	20·7369	1·7867	·0301606	25986
81	14824	·171542	18·6804	1·6142	·0271696	23477
82	14901	·171983	16·6186	1·4398	·0241707	20941

From D = 83° to D = 85° : $2\alpha = 60^m$.

83	·033575	·256955	21·6922	−2·8346	·0315501	−·0041227
84	33706	·257448	18 6043	2·4359	·0270589	35429
85	33823	·257867	15·5115	2·0344	·0225606	29589

From D = 86° to D = 87° : $2\alpha = 90^m$.

86	·075758	·381822	18·3573	−3·6426	·0266996	−·0052980
87	·075920	·382198	13·7717	2·7356	·0200301	·0039788

D = 88°, $2\alpha = 120^m$.

88	·13383	·499736	11·9986	−3·2131	·0174512	−·0046732

D = 89°, $2\alpha = 180^m$.

89	·29278	·707031	8·4852	−3·5141	·0123412	−·0051111

TABLE III. (*Corner Plates.*)

$D_2 - D_1 = 1°$: $A_2 - A_1$ variable with D_1

$D_1 = 0°$ to 36°, $A_2 - A_1 = \alpha = 4^m$.

D_1	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	F.	Circular Measure. C.	F.
0°	·000000	·000152	12·0012	11·9993	·0174550	·0174523
1	0	0457	11·9994	11·9975	·0174524	4497
2	0	0761	11·9939	11·9958	·0174444	4471
3	1	1066	11·9847	11·9939	·0174311	4444
4	1	1369	11·9720	11·9921	·0174125	4418
5	·000001	·001672	11·9555	11·9902	·0173886	·0174391
6	2	1976	11·9355	11·9885	·0173594	4366
7	3	2278	11·9117	11·9867	·0173248	4340
8	3	2580	11·8844	11·9850	·0172852	4315
9	4	2881	11·8534	11·9832	·0172401	4289
10	·000005	·003181	11·8188	11·9815	·0171898	·0174264
11	6	3480	11·7807	11·9798	·0171343	4239
12	7	3778	11·7389	11·9781	·0170736	4215
13	8	4074	11·6936	11·9765	·0170076	4191
14	10	4370	11·6447	11·9749	·0169365	4168

D_1	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C	F.	Circular Measure. C.	F.
15°	·000011	·004664	11·5922	11·9733	·0168602	·0174144
16	12	4957	11·5363	11·9717	·0167788	4121
17	14	5248	11·4767	11·9702	·0166922	4099
18	15	5538	11·4137	11·9687	·0166006	4077
19	17	5826	11·3473	11·9673	·0165040	4057
20	·000019	·006112	11·2773	11·9658	·0164022	·0174036
21	20	6397	11·2040	11·9644	·0162955	4015
22	22	6679	11·1272	11·9632	·0161839	3997
23	24	6960	11·0471	11·9619	·0160673	3979
24	26	7238	10·9635	11·9607	·0159457	3961
25	·000028	·007514	10·8766	11·9594	·0158194	·0173943
26	30	7788	10·7865	11·9583	·0156883	3926
27	32	8059	10·6930	11·9573	·0155523	3911
28	35	8328	10·5962	11·9562	·0154116	3896
29	37	8594	10·4963	11·9552	·0152663	3882
30	·000039	·008858	10·3932	11·9542	·0151163	·0173867
31	41	9119	10·2868	11·9534	·0149616	3855
32	44	9378	10·1774	11·9526	·0148024	3844
33	46	9633	10·0648	11·9518	·0146387	3832
34	48	9886	9·9492	11·9511	·0144705	3822
35	·000051	·010135	9·8306	11·9504	·0142980	·0173812
36	54	·010382	9·7089	11·9500	·0141210	3805

$D_1 = 27°$ to $D_1 = 48°$: $A_2 - A_1 = 5^m$.

27	·000051	·010074	+ 13·3663	+ 11·9330	·0194405	·0173559
28	54	·010410	13·2454	11·9313	·0192646	3534
29	58	·010744	13·1204	11·9298	·0190829	3512
30	61	·011073	12·9915	11·9284	·0188953	3491
31	65	·011400	12·8586	11·9271	·0187020	3472
32	·000069	·011723	+ 12·7218	+ 11·9258	·0185031	·0173453
33	73	·012042	12·5811	11·9245	·0182984	3435
34	76	·012358	12·4365	11·9234	·0180882	3419
35	80	·012670	12·2882	11·9225	·0178725	3405
36	84	·012978	12·1361	11·9216	·0176513	3392

D_1	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	Réseau Intervals. F.	Circular Measure. C.	Circular Measure. F.
37°	·000088	·013282	+ 11·9804	+ 11·9207	·0174248	·0173380
38	92	·013582	11·8210	11·9201	·0171929	3370
39	96	·013877	11·6580	11·9194	·0169558	3361
40	100	·014169	11·4914	11·9189	·0167136	3353
41	104	·014456	11·3214	11·9185	·0164663	3347
42	·000109	·014739	+ 11·1479	+ 11·9181	·0162139	·0173342
43	113	·015018	10·9710	11·9179	·0159566	3339
44	117	·015292	10·7907	11·9178	·0156945	3337
45	121	·015561	10·6072	11·9178	·0154275	3337
46	125	·015826	10·4204	11·9179	·0151559	3338
47	·000129	·016085	+ 10·2305	+ 11·9181	·0148797	·0173341
48	134	·016340	10·0375	11·9184	·0145989	3346
		$D_1 = 48°$ to $D_1 = 59°$: $A_2 - A_1 = 6^m$.				
48	·000192	·019608	+ 12·0448	+ 11·8823	·0175184	·0172821
49	198	·019907	11·8094	11·8828	·0171761	2828
50	204	·020201	11·5706	11·8835	·0168287	2839
51	210	·020488	11·3281	11·8843	·0164760	2850
52	216	·020770	11·0822	11·8853	·0161184	2864
53	·000221	·021045	+ 10·8330	+ 11·8864	·0157559	·0172880
54	227	·021313	10·5804	11·8876	·0153885	2898
55	233	·021575	10·3245	11·8890	·0150164	2918
56	238	·021831	10·0656	11·8906	·0146398	2942
57	244	·022079	9·8036	11·8923	·0142588	2966
58	·000249	·022321	+ 9·5386	+ 11·8941	·0138734	·0172992
59	254	·022557	9·2707	11·8959	·0134837	3019
		$D_1 = 59°$ to $D_1 = 63°$: $A_2 - A_1 = 8^m$.				
59	·000452	·030074	12·3603	11·8145	·0179773	·0171834
60	461	·030378	11·9994	11·8180	·0174524	·0171886
61	470	·030674	11·6348	11·8219	·0169221	·0171942
62	479	·030960	11·2666	11·8259	·0163866	·0172001
63	488	·031236	10·8950	11·8302	·0158462	·0172063
		$D_1 = 63°$ to $D_1 = 66°$: $A_2 - A_1 = 9^m$.				
63	·000617	·035139	12·2565	11·7850	·0178263	·0171406
64	628	·035439	11·8347	11·7906	·0172129	·0171488
65	638	·035728	11·4094	11·7965	·0165943	·0171573
66	648	·036007	10·9806	11·8026	·0159706	·0171662

$D_1 = 66°$ to $D_1 = 70°$: $A_2 - A_1 = 10^m$.

D_1	$(1-\cos\theta)$	$\sin\theta$	Réseau Intervals. C.	Réseau Intervals. F.	Circular Measure. C.	Circular Measure. F.
66°	·000800	·040006	12·2001	11·7561	·0177443	·0170986
67	812	·040303	11·7199	11·7641	·0170459	·0171101
68	824	·040588	11·2362	11·7722	·0163424	·0171219
69	835	·040861	10·7491	11·7806	·0156339	·0171342
70	845	·041121	10·2587	11·7894	·0149206	·0171469

$D_1 = 70°$ to $D_1 = 74°$: $A_2 - A_1 = 12^m$.

70	·001217	·049340	12·3090	11·6965	·0179027	·0170118
71	1232	·049637	11·7167	11·7093	·0170413	·0170300
72	1246	·049919	11·1210	11·7225	·0161749	·0170497
73	1259	·050185	10·5220	11·7361	·0153036	·0170695
74	1272	·050437	9·9194	11·7500	·0144272	·0170897

$D_1 = 74°$ to $D_1 = 78°$: $A_2 - A_1 = 15^m$.

74	·001988	·063031	12·3967	11·6093	·0 180303	·016885
75	2007	·063327	11·6402	11·6313	·0169300	·0169170
76	2024	·063602	10·8802	11·6539	·0158246	·0169499
77	2041	·063859	10·1169	11·6770	·0147144	·0169835
78	2056	·064095	9·3505	11·7004	·0135997	·0170175

$D_1 = 78$ to $D_1 = 82°$: $A_2 - A_1 = 20^m$.

78	·003655	·085416	12·4607	11·4671	·0181233	·0166782
79	3680	·085705	11·4356	11·5093	·0166324	·0167396
80	3702	·085968	10·4070	11·5521	·0151364	·0168018
81	3723	·086205	9·3753	11·5955	·0136358	·0168649
82	3741	·086416	8·3407	11·6393	·0121310	·0169286

$D_1 = 82°$ to $D_1 = 86°$: $A_2 - A_1 = 30^m$.

82	·008410	·129424	12·4917	11·1886	·0181684	·0162731
83	8447	·129699	10·9384	11·2880	·0159092	·0164177
84	8477	·129935	9·3818	11·3882	·0136453	·0165635
85	8502	·130132	7·8224	11·4892	·0113773	·0167103
86	8524	·130290	6·2608	11·5907	·0091059	·0168580

The Cambridge Machine for Measuring Celestial Photographs.

By Arthur R. Hinks, M.A.

As a preliminary to the preparation, some three years ago, of plans for a new measuring machine for the Cambridge Observatory, the Director authorised me to prepare an account of the methods which had at that time been proposed and of the machines which had been made for measuring celestial photographs. Some notes from this account may serve as an introduction to the description of our new machine.

1. *Types of Modern Measuring Machines.*

Modern machines for measuring rectangular coordinates may be divided broadly into two types :

Type A, in which positions upon the photographic plate are referred to a standard scale on the measuring machine.

Type B, in which a *réseau* impressed upon the plate takes the place of the standard scale, and it is necessary only to measure the place of the star image within a *réseau* square.

There are two leading forms of machines of type A.

A 1, the Repsold machine described by Bakhuyzen (*Bull. Com. Perm.* I. 164), in which the microscope tilts on its chariot, and is pointed successively on the plate and on the standard scale.

A 2, various machines, like the Zeiss "Comparator," in which there are two microscopes rigidly connected together, one to view the plate, the other to view the standard scale.

It is required in both these forms that the main slides shall be geometrically perfect, or that their errors shall be investigated and applied ; that the temperature corrections of the scale and plate shall be determined or in some way eliminated ; and it is further required in form A 1 that the tilting motion of the microscope shall be perfect. These are all serious impediments. Moreover, it is almost universally admitted that to control the distortion of the gelatine film it is necessary to impress upon the plate a *réseau*. To refer the *réseau* lines as well as the star images to a standard scale, so that the distortions of the film are actually determined over the whole plate, is cumbrous. Historically this is the original use of the *réseau*. But it was quickly realised that its true use is to treat it as the standard scale. This proceeding straightway eliminates all necessity for considering film distortion except within the one *réseau* square in which the star image to be measured lies. Within this square the distortion must be considered uniform, showing itself as error of run (after correcting for errors of division of the *réseau*), and

eliminated in the usual way when the star image is referred to the four sides of the square which enclose it.

For the measurement of plate impressed with a *réseau* whose errors are known or which may be neglected, it is sufficient to use a machine of type B. Of such machines there were three years ago two principal forms :—

B 1, the machines used at Paris, Potsdam, and elsewhere, consisting of a microscope with field of view sufficiently large to take in a R-square (5 mm.) and a filar micrometer in the eyepiece to measure the distance of the star image from the sides of the square.

B 2, the machines used at Greenwich and Oxford, in which these distances were measured by estimation on a finely divided glass scale in the eyepiece.

Both these forms possess structural advantages over machines of type A. The principal slides which carry the plate under the microscope need not be very accurate. They can produce no errors in measurement unless they are so bad as to tilt the plate sensibly.

On the other hand, machines of type B 1 have one grave disadvantage, which is shared in some degree by all machines with micrometer screws. They involve a great deal of turning of the screws, which wastes time and wears out the screws. In the Potsdam machine, for example, the screw makes about 1,500 revolutions per hour. A complete measure requires in general about twenty turns, ten forward and ten back.

Machines of type B 2 have the advantage that an invariable glass scale is used instead of a screw ; but the last place in the readings is only an estimate of the proportion in which the scale interval is divided by the image of a R-line. The accuracy with which this estimate can be made is shown to be only just enough for the purposes of the Astrographic Catalogue, when something must be sacrificed to speed ; but it is not enough for work of any greater refinement.

2. *The Principle adopted for the Cambridge Machine.*

Since the *réseau* is always used at Cambridge the machine was to be of type B. I proposed to try a combination of the essential features of the two machines of this type described above, and to use a divided glass scale moved in the microscope by a micrometer screw. In this method the chief part of the distance to be measured is read off on the scale ; the fractional part of the scale space is not estimated, but measured by the screw, which carries the results to one more place of decimals. Since it is easier to place a star disc or a R-line symmetrically between two divisions than bisected by one, the spaces of the scale should be numbered instead of the divisions. The screw turns in a fixed nut, and its point bears against the frame

carrying the scale. The head of the screw should be divided so that readings of the head increase as it is turned to make the scale readings on the object diminish ; the sum of the readings upon a fixed object of scale and screw, reduced to the same units, then remains constant.

Fig. 1.

The method of making a measure on this plan is illustrated by the annexed diagram (fig. 1), which is drawn to represent the proportions which were adopted.

The R-interval is 5 mm. ; 100 spaces of the scale = 1 R-interval ; and the pitch of the micrometer screw is $0^{mm}{\cdot}5$ = 10 scale spaces.

The R-square is brought into the field, and it is required to measure the distance of the star S from the line A.

The scale is moved by the screw until the nearest space is bisected by line A.

Scale reading 52, screw .349 say (fig. 1*a*).

The scale is then moved so that the star is placed symmetrically on an adjacent space.

Scale 118, screw .502 (fig. 1*b*)

And finally the line B is made to bisect a space.

Scale 152, screw .341 (fig. 1*c*)

The zero is of course perfectly arbitrary.

Now these numbers expressed as decimals of a R-interval are :

$$\left.\begin{array}{lll} A & 0{\cdot}52 & 0{\cdot}0349 \\ S & 1{\cdot}18 & {\cdot}0502 \\ B & 1{\cdot}52 & {\cdot}0341 \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

And adding corresponding readings of screw to scale we have :

$$\left.\begin{array}{ll} A & 0{\cdot}5549 \\ S & 1{\cdot}2302 \\ B & 1{\cdot}5541 \end{array}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

It follows that the run is ·0008, correction positive, and the corrected distance of star from A is

$$\cdot 6753 + \cdot 0005 = \cdot 6758.$$

And it should be noted that the result is the same, whichever spaces are bisected, so long as the ratio of screw to scale is correct. We might have used different spaces and obtained the readings—

50	549
119	402
154	141

and these would reduce as before to the numbers of (2).

This arrangement seemed to possess the following advantages. The whole length of screw used in making the measure need never be more than one or two tenths of a revolution ; there is therefore no time lost in running the screw backwards and forwards, and there is very little wear. It is scarcely possible that progressive error of the screw can come in ; and since repeated settings should be made on different spaces, and consequently with different parts of the screw, periodic error would always tend to go out.

Finally, if it is desired to make rough preliminary measurements, the screws may be left untouched, and measures made by estimation on the scale, as in the Greenwich and Oxford machines.

3. *Details which are desirable.*

In order to be able to measure in both coordinates at once, it is desirable that the scale in the microscope should consist of two scales crossing at right angles and moved by two screws at right angles. Further, to obtain the full power of using the scale for estimation in the Oxford method, the eyepiece must command a field two R-intervals at least in diameter ; and this is best accomplished by putting it on a slipping piece.

The microscope objective which projects an image of the plate on the micrometer scale is best placed to give a magnification unity. This arrangement possesses the convenient property that the image of the R-square can be made to fit the scale, and the error of run reduced to zero by racking the objective in or out with respect to its tube. No further adjustment is required. The plate remains in focus on the scale for a considerable range of motion of the objective, since for magnification unity the distance between conjugate foci of the lens is a minimum. This arrangement of the objective seems to be in common use, but its advantage has sometimes been lost because the objective has not been given a power of adjustment independent of the microscope tube.

A reversing prism in the microscope, by means of which the field of view can be rotated, is very useful in avoiding personality in measurement. In my own case it is a necessity, because the astigmatism of my eyes makes it impossible for me to see vertical lines as well as horizontal. The power of readily turning the field until the divisions of the scale I am using are horizontal is then a great convenience.

It is a well-known inconvenience of working with a micrometer that in making the settings the eye has to be close down to the micrometer heads, and must be withdrawn to read the heads. A minor inconvenience is that the nose has frequently to be pressed flat against the micrometer box. These difficulties can be avoided by using a compound microscope of low power instead of an eyepiece to view the micrometer scale. The divided heads are then further away from the eye, and in a position more convenient to handle. With the present machine it is easy to read the head with one eye without removing the other from the eyepiece.

It is an immense convenience if all adjustable parts are furnished with divided scales.

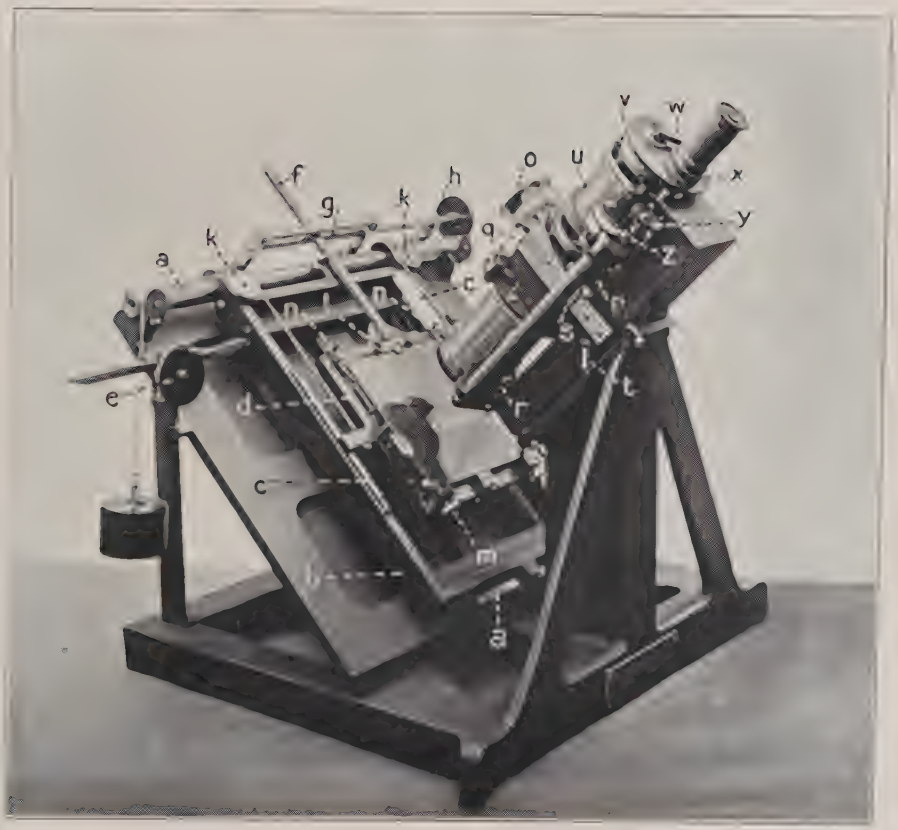
4. *The Construction of the Machine.*

The construction of the machine upon these lines was approved by the Director of the observatory, and the Cambridge Scientific Instrument Company, Limited, undertook the work. A draft of the features outlined above was sent, and plans embodying them were prepared by Mr. Horace Darwin. It is a pleasure to acknowledge the great debt which is due to him for the careful thought he has given to the design. Every detail of the machine was thoroughly discussed, and I think that all difficulties have been successfully overcome. Thanks are due to Mr. Newall for help in many ways ; and several excellent ideas were borrowed from Sir David Gill's description of his new Repsold machine. Acknowledgment must also be made of the trouble which was taken by Messrs. Zeiss in arranging the optical parts, dividing the scale, and especially in designing for us a new form of reversing prism.

The accompanying photograph (Plate 12) shows the general appearance of the machine.

The base is a heavy iron casting resting on three feet, cast in one piece with the pillar which carries the microscope. Upon the base is erected a cast-iron frame which carries the main horizontal slides *a*, *a* of cylinder and plane form. On these rest a frame *b*, which carries the main vertical slides *c*, *c* of similar form, and on these slides rests the plate-carrier *d*.

The arrangements for giving motion to the plate-holder, and for counterpoising it on the vertical slide, demand a word of explanation. The frame *b* is moved along the horizontal slide by means of the rack and pinion *e*. The plate-holder carries a rack



THE CAMBRIDGE MACHINE FOR MEASURING CELESTIAL PHOTOGRAPHS

"collecting lens." Its function is to converge the axes of the pencils of light diverging from all points of the glass scale, without altering appreciably the angle of divergence of the beams in each individual pencil. This is necessary in order that the whole of the pencils should be brought into the objective of the view microscope.

Objective II., at 5, is a Zeiss "symmetric aplanat" combination of 35 mm. focal length. Immediately above it is the reversing prism 6, which is of a new form, devised by Messrs. Zeiss. The usual place for a reversing prism is outside the eye-lens. When it is placed there it cuts down the field of view very considerably, as is well known, and this was inadmissible. The new reversing prism has three reflecting surfaces, and it is an irregular solid very hard to figure. But its action may be

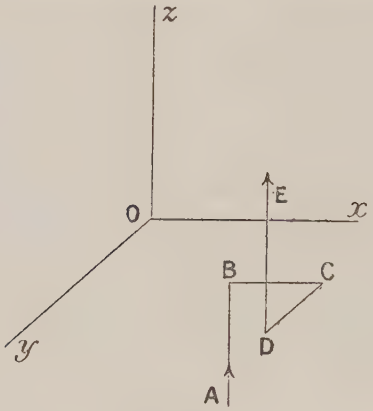


Fig. 3.

described thus : Let Ox, Oy, Oz be a system of rectangular axes. Let AB be the axis of the central pencil passing through objective II. It enters the prism at a face parallel to the plane xy. At B it is reflected parallel to the axis of x ; at C parallel to the axis of y ; and at D parallel to the axis of z ; so that it emerges along DE parallel to its original direction AB, through a face parallel to the face of entry. If these faces are large enough to take the diverging beam from the objective there is no loss of field of view ; and the definition of the microscope does not seem to be sensibly impaired by the action of the prism.

Since the optical axis is displaced parallel to itself, the eyepiece 7 must be mounted eccentrically on the end of the tube.

To return to the details of the mounting of these optical parts :

The objective 1 is mounted in a steel tube with four projecting studs, which bear outside a smaller steel tube fixed inside

the body of the microscope and turned concentric with it (see fig. 2). A spring keeps the two together, and a screw with divided head (seen reflected in the glass plate in the photograph) slides the objective tube along the other tube. This gives the motion of the objective required to fit the *réseau* square to the scale.

The double rectangular slide on which the divided scale is moved by the two micrometer screws is of a new pattern. It is geometrically desirable that the plane of the scale should coincide as nearly as possible with the plane of the slides, and this is difficult to manage if the slides are of the ordinary pattern, one crossing over the other.

The principle of the new slide is illustrated in fig. 4. The scale is held in a square block of steel, with two sides, AB, AC, ground accurately at right angles to one another, and to the face of the block. The points of the micrometer screws bear on these sides. Below the block is screwed a piece of steel forming two

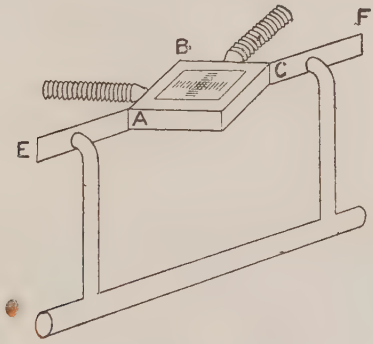


Fig. 4.

wings, AE, CF, with faces worked to a plane through the diagonal AC of the block, and perpendicular to its face. At some distance below is an axle working in V bearings parallel to AC ; and from this axle spring two arms rigidly attached to it, which come up and bear against the faces of the wings. Spiral springs are wrapped round the axle, and the block is held against the points of the screws by the pressure of the arms upon the wings.

When either micrometer screw is turned the block is moved in a direction along the axis of the screw, and at the same time it is compelled to keep its diagonal AC always parallel to the axle from which the arms spring. The block has therefore motion in the plane ABC only in the two directions, mutually at right angles, of the axes of the micrometer screws.

Two long steel pins with sharp conical ends enter two shallow conical holes drilled in the under side of the block. The pins are driven upwards by spiral springs. This arrangement allows

ample lateral movement to the block, while it keeps its face ABC pressed against the worked inner surface of the top of the micrometer box, and it reduces friction of the slide to a minimum.

This new form of double micrometer slide is perhaps the most beautiful of the many pieces of ingenious geometrical design which Mr. Darwin has put into the machine. It is impossible to do justice to them in a brief sketch.

The micrometer screws *o* are of half-millimetre pitch. They were made by Messrs. Brown & Sharpe, of Philadelphia, U.S.A. The heads are of white celluloid divided into one hundred parts. Since the nut is fixed in the side of the micrometer box, and the motion transmitted by the point of the screw, the divided head has a considerable range of motion in the direction of its axis. The fiducial marks consist therefore of fine blackened silver wires stretched in frames and brought close down to the surfaces of the heads.

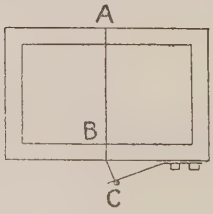


Fig. 5.

It is frequently required to mount fine wires in tension, and the ingenious method of fixing these wires used in this machine is worth describing. V-shaped cuts are made in the frame at A and B (fig. 5). C is a small piece of spring screwed to the side of the frame, and its end, which lies rather below the plane of AB, is pierced. A knot is tied in the wire ; the wire is passed through the hole in C and laid in the cuts B and A ; it is pulled up a little against the spring and fastened with a drop of solder at A. This arrangement keeps the wire always in tension.

The divided glass scale rests on three small projections in the edge of the square hole cut through the steel block. It is provided with an adjustment in orientation by which the scales may be set parallel to the axis of the micrometer screws. The complete glass scale will consist of two similar scales crossing at right angles. Up to the present Messrs. Zeiss have not been able to furnish an entirely satisfactory scale of this pattern, which seems to be difficult to make. The first crossed scale which they supplied consisted of two scales divided separately on thin glass and cemented face to face. This form was objectionable. It is essential that no thickness of glass shall be interposed between the scale and the objective, which projects on it the image of the plate. Otherwise there is introduced away from the centre of the field the distortion due to oblique passage through a parallel

plate, or perhaps to a plate not quite parallel. It did not seem worth while to incur the necessity of investigating and applying a troublesome small correction. The parts of the scale were therefore separated, and a single one is being used at present. Messrs. Zeiss hope to be able before long to supply the crossed scale ruled on one piece of glass.

The "collecting lens" in a steel cell is screwed into the top of the micrometer box q, so that it comes down very close to the scale.

This completes the description of the lower section of the microscope. The whole is free to turn in the lower pair of V's, r, r of the microscope cradle, through an angle of 90°. At the limits of its range of rotation it is brought up against stops s, adjusted so that the screws are parallel to the main slides, and spiral springs, t, can be hooked on to hold it in those two positions. It is possible therefore to measure the coordinates increasing with R.A. and Decl., or by turning through 90° with N.P.D. and R.A. The photograph shows the latter position. This power of rotation is of the greatest value in testing the centering of the somewhat complicated optical system.

The upper section of the microscope, with which the scale is viewed, consists of a low-power compound microscope, for the reasons outlined above. It is mounted in a steel tube u of the same diameter as the lower section of the microscope. Objective II. is screwed centrally into the lower end. Immediately above it is the reversing prism on a plate mounted on three screws by which the prism can be squared on to the optical axis. It should be remarked that this adjustment is not effective in the plane which contains the axis of the incident and emergent beams (the plane of fig. 2). A tilt of the prism in this plane produces no effect upon the direction of the emergent beam, which is controlled entirely by the angles at which the reflecting faces of the prism are worked.

The eyepiece plate v is double. A pin through the upper plate passes through an oval slot in the lower, and thence into a large flat washer. An arm w is attached to a nut on the upper end of this pin, and by giving it a slight turn the washer is tightened and the two plates clamped together. This makes a convenient slipping piece. If the eyepiece is held between the thumb and forefinger, the arm can be turned with the little finger and the nut loosened, the eyepiece slipped to the required position, and the whole clamped again.

There are three eyepieces which give powers, 20, 40, and 60 on the microscope. They are mounted on flanges x which have screws cut away like a gun breech, so that they slip home, and a slight turn holds them tight. Measuring is done with the power 20; but if any photographic defect in the image is suspected a higher power can be slipped in already adjusted to focus, and the suspected image examined.

The upper tube can be completely rotated in its V's. A turn through 45° turns the field of view through 90° by virtue of the

reversing prism. A weak spring catch holds the tube lightly in the eight positions in which the scale and *réseau* lines appear in the microscope horizontal and vertical.

There are three separate adjustments for focus. The milled head y slides the upper tube along its V's with respect to the lower, and brings the divided scale into focus in the eyepiece. The divided head z slides both tubes together along their V's, and brings the plate into focus on the scale. Finally, the divided head seen reflected in the plate gives an independent motion to objective I. and corrects any error of runs.

The two latter heads are divided on celluloid into one hundred parts: the first, because it is difficult to focus the plate on the scale with extreme accuracy at a single setting: a number of settings are made, and the head left at the mean reading; the second head was divided so that a given error of run could be corrected immediately without a series of trials.

This completes the description of the microscope.

The setting scales on the main slides (one of them seen through the plate in the photograph) are of white celluloid, divided to mm., and numbered in R-intervals. The numbers run both ways, one set in black, the other in red, so that a star can easily be found in either the direct or reversed position of the plate. The indexes are adjustable.

Some care has been taken to shield the measurer from all unnecessary light. The instrument stands on a table covered with black cloth. A black curtain hangs immediately behind the machine, with only a small hole in it to admit light to a reflector beneath the plate. Another black curtain hangs to the left. The greater part of the machine is painted dead black. These precautions make it possible to keep open the eye which is not in use; no light falls on it to dazzle it, and there is nothing bright in sight to attract its attention. The comfort and absence of strain on the eyes are very marked.

The essential parts of the machine are made of steel or cast iron, very massive. The machine is little sensitive to temperature, and its rigidity is remarkable.

5. *Rapidity and Accuracy of Measurement.*

It is as yet a little early to estimate definitely the rapidity with which measures can be made with this machine. The plates which have so far been measured with it have been measured in different ways for experimental purposes, and no long series of plates has as yet been treated in a systematic way. At present I have adopted the plan of making one pointing on each of two R-lines and two pointings on the star image for each coordinate. Working in this way, and recording one's own measures, it will, I think, be easily possible to measure twenty-five stars per hour in both coordinates.* This is of course much

* [Note added later.] Further practice has increased the rapidity to thirty stars per hour.

less than Sir David Gill's record, eighty stars per hour, including diameters, though it must not be overlooked that it takes two observers to do this, one recording for the other. When allowance is made for this the Cape machine is probably getting on for twice as fast, and it will be well to attempt to show that increased accuracy compensates in the Cambridge machine for diminished rapidity. On the other hand, the Cambridge machine is a great deal faster than those used at the Paris and other French observatories, where they measure from twelve to fifteen stars per hour.

It may be mentioned as a matter tending to rapidity that there is no need to record the measures in all the detail given in the example (Section 2). For instance, the scale and screw readings may be added together before the figures are set down ; there is no need to take note for the second R-line of more than the last two figures of the micrometer reading ; and even these need not be set down—they may be compared with the reading on the first, and the run set down directly. And since the mean run is easily reduced to nearly zero, the run of individual squares will rarely amount to ten units in the last place, and the necessary correction may be determined by inspection (see the example later in the discussion of the Cape machine).

The proportions of screws and scales were designed on the supposition that $0^{mm}\cdot0005$ is just less than the smallest amount of shift between two well-defined marks which can be appreciated by the eye with a microscope magnifying twenty diameters. This should make it just not possible to set continuously on a well-defined point to the same estimated figure on the micrometer head, but the range of variation should not be more than a few units in the estimated place. Some experience in determining division errors has shown that this estimate is certainly not too high. The probable error of a single setting on a perfectly defined mark is not so high as $\pm0^{R}\cdot0001$ ($=0''\cdot017$ on the plate) when the observer is in good form. This is high testimony to the stability and workmanship of the machine ; but it is of course no criterion as to the accuracy which may be expected in measures of star plates. Professor Kapteyn has given in *Publications of the Astronomical Laboratory of Groningen* (pt. i. p. 81) some interesting figures to show that the real accuracy in the measured position of a star image is much less than the accuracy estimated from the divergence of successive pointings on it, or even from the agreement of measures made in reversed positions of the plate ; that is to say, the position of a star image has a real error of its own, independent of the errors of measurement, due to such causes as irregularities in the size and arrangement of the silver particles. The true P.E. of a measured position on the plate, deduced by comparison with the results of other plates, is therefore not a fair test of the capabilities of the machine ; it depends more upon the defining power of the telescope, atmospheric disturbances, &c., as well as the quality of the plates.

The accuracy of the measuring machine is sufficient for its purpose if the errors in pointings due to the machine are much less than those due to irregularities in the star image.

Judged by this standard the Cambridge machine is amply accurate. A first determination of the P.E. of a complete measure (one setting on each R-line and two on the star in each position of the plate) derived from a direct reduction of one plate to another by means of eleven common stars, is $\pm 0^{R}\cdot0004$, $=0''\cdot07$; whereas the P.E. of a *single setting* on a well-defined mark is less than $\pm 0^{R}\cdot0001$. It is to be concluded that no gain in accuracy would result from making the micrometer screws finer.

6. *Errors of Screws and Scales.*

a. Progressive Error of Screw, and Ratio of Screw to Scale.—A well-defined speck was chosen on the plate, and every tenth division of one of the scales from 150 to 50 was set upon it in turn by the screw, the plate remaining fixed. The values of successive distances of ten-scale spaces in terms of the screw revolution were found to be $1^{r}\cdot003$, 3, 2, 3, 2, 2, 2, 1, 2, 1 ; the small variations in these distances include of course division errors of scale as well as progressive errors of screw. There is evidently no appreciable progressive error in the screw over ten revolutions unless it should chance to be exactly compensated by a similar error in the scale ; and this is shown later not to be the case. The pitch of the screw differs from that of the scale by about two-tenths per cent., which is negligible, since only fractions of a screw revolution are used.

b. Division Errors of the Scale.—These were partially examined by the method used by Sir David Gill in his investigation of the heliometer scales (*Monthly Notices*, xlix. p. 110). An etched scale by Zeiss was mounted on a plate and used as the auxiliary scale. The space between 150 and 50 was taken as correct, and the error of every tenth intermediate space found. The means of two determinations, which agreed well, gave for the corrections -12, $+4$, $+9$, $+3$, $+3$, $+17$, $+13$, $+32$, $+8$, all multiplied by 10^{-5} R-intervals. Only one of these is at all large, and that is due to an obviously bad cut of one division, which would be avoided in use.

A similar examination of the error of every alternate space from 110 to 90, assuming the errors for these spaces found above, gave for the intermediate space errors $+9$, $+12$, $+14$, $+3$, -3, -4, -8, $+13$, $+6\times10^{-5}$ R-intervals.

It is evident that the division errors of the scale examined rarely amount to more than one in the last place of decimals used ($\cdot0001$ R$=\cdot0005$mm.). It seems safe to treat these errors as accidental.

c. Periodic Errors of Screws.—An examination was made of the screw used with the single scale at present in use. No periodic term could be detected having a coefficient so large as a

single thousandth of a screw revolution. If there are periodic errors they are so small that they could not be determined without using a reading microscope to read the divisions of the head to a greater accuracy than estimation of tenths gives. The screw may therefore be pronounced perfect.

7. *Comparison between the Cambridge Machine and Sir David Gill's Repsold Machine.*

It is not fair to compare directly the performances of the Cape and Cambridge machines, because the former was designed for the measurement of the astrographic catalogue plates, and something is admittedly sacrificed to speed. On the other hand the Cambridge machine was made for measuring plates for parallax determinations, and the highest attainable accuracy is the first consideration. It is, however, admissible to suggest that it seems probable that the principle of the Cambridge machine could be adapted to the Cape work, with a gain in accuracy, and no loss of speed.

Professor Turner has brought forward several objections to the Cape machine (*Monthly Notices*, lix. p. 135, 1899 Jan.). The weightiest is the conclusion that on Sir David Gill's own showing (*Monthly Notices*, lix. p. 73, 1898 Dec.) the micrometer screws of his machine should be already worn out. It seems to me that this estimate is exaggerated. It is based upon a comparison between the microscopes of the meridian circle and the measuring machine. In the former, if a complete determination of run is made for each reading, the screw must be turned through a range of 5 rev. backwards and forwards each time. In the latter the screw is turned only from star to star within the R-square, and is not set upon the R-lines themselves. The criticism appears, however, to apply precisely to the use of the screws in the Paris and Potsdam machines. In any of the cases the wear of the screw soon spoils the accuracy of the machine.

A second objection which Professor Turner brought forward is the partial neglect of error of run. Sir David Gill gives reasons for believing that its effect is small. Reference to his paper, p. 67, shows that he adjusts his square of webs to fit the mean R-square on the plate, and that if any individual R-square does not fit the webs he makes a symmetrical pointing on it; the position of the star image should thus be referred to the middle of the square, and the effect of run halved. The efficacy of this procedure seems to me somewhat doubtful. In placing the R-square on the square of webs attention must be concentrated on one line or the other, and the accuracy of setting on the one is considerably greater than the power of perceiving afterwards a want of symmetry in the other—at least, it seems so to me. It is probable that on this account, when the runs are small, the setting is made on one line, and the symmetry of the other is regarded as sufficient when it is really not so. The

error of run would then come in to its full amount. This remark does not of course apply to large errors of run, which are readily enough perceived, but to the continual small changes, due to distortion of film, errors of R-lines, and want of flatness of plate, which are evident to measurement, but scarcely apparent to inspection.

I venture, with some diffidence, to suggest that the principle of the combined scale and screw might be used in the Cape type of machine with very little, if any, loss of speed and some gain in accuracy. At the Cape Observatory the measurers work in pairs, one acting as clerk to the other. Suppose the measures made exactly as I have described in paragraph 2, and let us take the figures used in that example. The observer makes the setting on the first R-line and calls out 52,349; the recorder enters this at once as 5549. The observer now sets on the other R-line with results 152,341; the recorder need only do the simple subtraction of the last figures in his head, 49—41 = +8, and record "run +8." Finally the observer sets on the star with readings 118,502. The recorder enters these as 12302, and while the observer is finding the next star performs the subtraction 12302—5549=6753, adds 5 for run, and writes down the corrected result ·6758. The observer has to make one more setting per star, three instead of two; but on the other hand he saves the time spent in giving several turns to the screw. The recorder has more work to do, but I think he could keep pace with the observer after a little practice. The result should be more accurate, since runs would be rigidly allowed for; the screw would not wear out nearly so quickly, and there would be no anxiety about permanence of somewhat complicated adjustments.

The rapidity of measurement which might, I think, be attained with the Cambridge machine used in this way depends of course on the condition that two people are available for the work. It is rather a special case which was not contemplated in the work for which the machine was designed. But the very satisfactory results which the combination of screw and scale has given at Cambridge seem to be some excuse for entering rather at length into the question whether their use might not be the means of avoiding one or two of the objections which have been raised to the methods adopted at the Cape Observatory, and more recently at Sydney and Melbourne, for the measurement of the Astrographic Catalogue plates.

I am indebted to the Cambridge Scientific Instrument Co. for the negative from which the plate illustrating this paper was prepared.

Cambridge Observatory: 1901 *May* 3.

Note on the Geometry of the Siderostat. By H. C. Plummer, M.A.

1. In the following note I have collected some of the principal geometrical propositions which relate to the siderostat and the motions connected with that instrument. There is little that can be considered essentially new in this account, but at the same time it may not be altogether useless to attempt to put the theory in a simple and clear light. Hitherto the siderostat has engaged the attention of the physicist rather than of the astronomer, and it is in France especially that its development has been promoted, while in this country it has been comparatively neglected. For these reasons there is perhaps some excuse for considering the matter in this place.

2. The principal propositions to which reference is made, and which are nearly all quite well known, are these :

1. The normal to the mirror describes a cone of the second order.

2. The circular sections of this cone are (*a*) perpendicular to the axis of the Earth, and (*b*) perpendicular to the ray reflected in a fixed direction.

3. The section (*a*) is described with uniform motion.

4. The section (*b*) is described with motion derived by stereographic projection from uniform motion.

5. The intersection of the mirror with the plane of reflexion describes a cone of the second order.

6. The circular sections of this cone are (*c*) perpendicular to the axis of the Earth, and (*d*) perpendicular to the ray reflected in a fixed direction.

7. The section (*c*) is described with uniform motion.

8. The rotation of the field is precisely the same as the motion with which the section (*b*) is described.

3. In the figure (fig. 1) which illustrates this note all directions are referred to a sphere of which the centre is C, the fixed centre of the siderostat mirror. The instrument is so adjusted that the star S, whose N.P.D. is δ, is reflected in the fixed direction CA. Hence the normal to the mirror CN bisects the arc SA. The pole is P, and is reflected to Q, so that the arc PQ is also bisected at N. It is at once obvious that AQ is equal to PS, and that Q therefore describes a circle round A of angular radius δ. The lines ACA′, PCP′ are diameters of the sphere. The intersection of the mirror with the plane of reflexion is represented by CM. This line is perpendicular to CN, and lies in the plane ASA′, which is clearly the plane of reflexion.

4. All the geometrical properties enumerated above become evident when the figure is examined. The lines A′S and CN are parallel for $SA'A = \frac{1}{2}SCA = NCA$. Similarly P′Q and CN are parallel for $QP'P = \frac{1}{2}QCP = NCP$. Hence the three

lines A′S, P′Q, and CN, being parallel, generate cones which are similar and similarly placed. It follows that parallel sections of all three cones are similar. But the cone whose vertex is at A′ has a circular section which is uniformly described in a plane perpendicular to CP, and the cone whose vertex is at P′ has a circular section in a plane perpendicular to CA. Hence sections of the cones which are parallel to either of these planes are circular. Now the section perpendicular to A′A of the cone

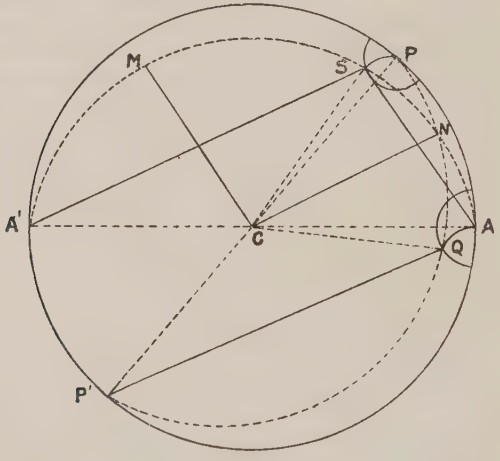


Fig. 1.

whose vertex is at A′ is the stereographic projection of the locus of S, which is the other circular section. We have thus obtained incidentally a fairly simple proof of the well-known theorem that the stereographic projection of a circle is a circle. But we have at the same time proved the propositions in § 2 numbered 4 and 8 as well as those numbered 1–3. The last proposition on the list is, as far as I know, new, and was obtained analytically before the possibility of so simple a geometrical proof was realised.

5. The remaining theorems refer to the motion of CM, and are derived with equal facility. For since CN is parallel to A′S, and CM is perpendicular to CN, CM is parallel to AS. Hence CM and AS generate cones which are similar and similarly placed, and the circular sections of the one are made by the same planes as those of the other. But the cone whose vertex is at A has for its sections the locus of S and the stereographic projection from A of this locus. The theorems 5–7 of § 2 follow at once.

6. The connection between the rotation of the field and the motion of the normal to the siderostat mirror suggests the possibility of deriving the mechanism for giving compensating motion to a photographic plate from the siderostat itself. The convenience

and advantage of such an arrangement are obvious, but there are also difficulties to be anticipated. If it proved that these could be overcome, an interesting solution of the problem of compensation would result. It may be pointed out that in the stereographic projection from A′ of the small circle which is the path of S, the projection of P is not of course the centre, but corresponds to the point B (fig. 1, p. 402), which represents the position of the fixed pin employed in the method I have suggested recently for compensating the rotation. Hence we have this theorem in stereographic projection: The plane A′SP cuts the circle S again in S′. The projections of S and S′ are S_1 and S_1'. If the circle S is described uniformly, S_1' describes the projected circle uniformly, while the motion of S_1 obeys the law

$$\tan \tfrac{1}{2}\theta = K \tan \tfrac{1}{2} nt$$

7. The quite simple description of the geometry of the siderostat given above will render clear the principles which underlie the construction of this instrument. In Silbermann's device CS and CA are produced backwards and materialised as bars, and a linkage is added to form a rhombus with the diagonal of which coincides a slotted bar which is normal to the mirror. In all other devices the properties given in § 2, 3, or 7 are employed, and accordingly two types have resulted. In the one class we have the mirror driven by a rod fixed normal to the mirror; in the other by a rod or bar fixed in the plane of the mirror. The former type seems to have been first realised by S'Gravesande, and may be considered by imagining the mirror at A′ instead of C. Then A′S will represent the direction of the normal, which in its material form must of course be reversed, as A′T (fig. 2). It is only necessary to make T describe a

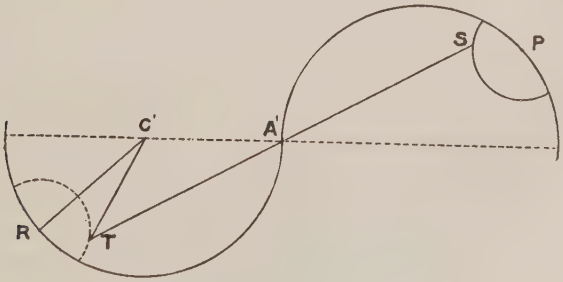


Fig. 2.

small circle about the polar axis RC′ on a sphere which passes through the centre of the mirror. The point T represents a ring through which the normal passes, and which can be set at the appropriate polar distance. The second type of instrument was devised by Gambey. If we imagine the mirror transferred to A,

it is easy to see, as in the other case, how the necessary motion is given to AS, the line in the plane of the mirror. But further the plane of the mirror must remain perpendicular to the plane ASA′. To secure this, that central line of the mirror which is perpendicular to the driving rod is constrained to remain perpendicular to CA, the direction of the fixed reflected ray. Thus the plane of the mirror contains a line which is perpendicular to two lines in the plane of reflexion, and the required condition is fulfilled. A small instrument, which has been found very useful, has been designed by Dr. Johnstone Stoney on this principle. The siderostat of Foucault combines some of the features of both types, the motion of the mirror being controlled partly by the normal rod and partly by a slotted bar in the plane of the mirror and passing over a rod which is the prolongation of TC′ (fig. 2). The bar is thus kept parallel to AS (fig. 1), as is easily seen, and provides the means of maintaining automatically the longer dimension of the mirror (if this is neither circular nor square) in the plane of reflexion.

The description of these instruments is of course too brief to convey a very satisfactory idea of the details of their construction. What is aimed at is to point out the relations of the different types. The principles outlined here will probably suffice to render the mechanism of any particular instrument immediately intelligible on inspection of the concrete example.

Oxford: 1901 May 9.

The Spectrum of Nova Persei. Note 4.

By the Rev. Walter Sidgreaves, S.J.

In note 3 on the spectrum of *Nova Persei* in the last number of *Monthly Notices* it was observed that at the times of minimum of the light curve the bright Hζ band was greatly extended on its violet side, and that one of the blue bands appeared to be too bright to be satisfactorily explained by the greater contrast of a weaker continuous spectrum.

The photograph of March 28 shows two prominent lines of greater brightness in the so-called extension : one at its red side adjoining, or perhaps overlapping, the hydrogen band ζ, and the other near its margin on the more refrangible side. The wavelengths of these two lines are found to be 3882 and 3859, referred to the centre of bright Hβ. The extension may therefore be a cyanogen band, composed of the nineteen lines measured by Lockyer * between 3883 and 3855, of which the strongest lines are 3883, 3871, 3862, and 3855 ; and it will be convenient to call it, provisionally, the CN band. This band and the most prominent of the blue bands at 4633 are the characteristic features of the spectrum at the minima of the light curve. Both bands either

* Watts, Index of Spectra 168, revised edition.

come into being at these times, or are completely absorbed at times of greater general brightness of the star, neither of them being accounted for by the greater contrast of a feebler continuous spectrum ; for at Hζ the continuous spectrum was never so bright as the CN band, and the blue band is not a previously existing band grown stronger, but it occupies the space between two previous bright bands, in the same manner as the CN band occupies the space between Hζ and Hη.

The dates on which this spectrum has appeared are indicated by the brackets in the following list, and may be called the minimum type : February 28, March 3, 7, 8, 12, 16, 17, 20, 21 (22), (25), 27 (28), April 1, 4, 5, 9 ? (11), (12), (13), (16), 19 (20), (21), 23 (25), (26). On the other dates not in brackets the CN band is completely absent, and the blue band 4633 is replaced by a pair of bands, one on each side of its position. And it is noteworthy that the pair of bands appeared for the first time on March 21, the day before the first photograph of the minimum type spectrum. They had formed from a very broad bright band covering about 5 $\mu\mu$, which had been a constant feature of the spectrum from February 28 to March 20.

The query written against April 9 in the above list of dates signifies uncertainty of the type of spectrum on that date. It seems to be in the state of transition from the non-minimum type to that of the minimum, or *vice versa* ; the CN band is absent, but the blue band 4633 is present in a diffusive state, and not strong, and without its otherwise constant companion at 4681.

The radiant energy of the CN band seems to be very great. On the later dates, notwithstanding the low altitude of the star and the sensibility curve of the plate, it is stronger than Hδ. At the same time a remarkable change appears in the relative strengths of Hε and Hδ. On the spectrographs of April 16, 20, 25, and 26, Hε is stronger than Hδ, and the CN band is stronger than Hε ; so that the relative intensities of these three bands are in the inverse order of the sensibility curve, and the relation is not exaggerated by expressing the density of the silver deposit at Hδ by d, and writing $2d$ and $3d$ for the densities at Hε and the CN band.

Stonyhurst College Observatory:
1901 *May* 5.

The Visual Spectrum of Nova Persei.

By the Rev. A. L. Cortie, S.J.

The following observations of the visual spectrum of *Nova Persei* were taken by means of an equatorially mounted refracting telescope with an excellent 5-inch object glass by Alvan Clarke, but without clockwork, and a McClean eyepiece star spectroscope of small dispersion. The estimates of the magnitude of the star during the times of the observations, after the star

became invisible to the naked eye, were made in the small finder of the instrument by Mr. Joseph Ronchetti. The observation of the star's spectrum on April 21 is also due to the same observer, and he assisted in all the other observations. The wave-lengths of the lines, which are given to three figures, do not pretend to any degree of exactness, but may be a guide to the identification of those lines the origin of which was not at once evident. They were derived by a comparison of eye estimates with the photographs of Father Sidgreaves, and that taken on February 28 by Professor Hale, and published in the Yerkes Observatory Bulletin No. 16. By this means it was quite easy to fix the approximate mean wave-lengths of the bright and dark bands observed in the small instrument employed. The details of the observations are as follows.

On the three nights, March 12, 16, 20, the predominant lines were the hydrogen lines α, β, and γ, the first two being very brilliant, a bright yellow line, presumably D, and a bright triplet in the green comprising the *b* group, and bands at λ 500 and λ 550. On the 12th a broad dark absorption band, slightly more refrangible than D, was seen, identified by Professor Hale (*loc. cit.*) as corresponding to D_3.

On these three nights the magnitude of the star during the times of observation between the hours of 9 and 11 P.M. was estimated as equal to δ, between ν and κ, and equal to ν—or 3, nearly 4, and 4. The colour on the 12th and 16th was noted as very like that of the planet *Mars*.

On the 21st the magnitude of the star was estimated as slightly less than ν, or fourth magnitude. The night was perfect for seeing, and no less than sixteen bright and dark bands and lines were observed. Of these, nine were bright lines, Hα, Hβ, D, *b*, λ 500, λ 466, a band still more refrangible, and two others in the red, the one less and the other more refrangible than Hα. The former of these is very probably the line λ 6678 due to helium, and so frequently observed in company with D_3 in solar prominences. It was again seen on March 28, when the star had fallen to the sixth magnitude. The same line is represented in the drawing of the visual spectrum of *Nova Aurigæ* made by Professor Campbell at the Lick Observatory on 1892 February 28.

The other very faint and fine bright line likewise seen in the red was estimated at one-third of the distance between C and D, somewhere about λ 640. This line was again seen on the 25th, was invisible on the 27th, and reappeared on the 28th, thus becoming visible when the star fell in magnitude to 6, and disappearing with the intermediate rise of the star to the fourth magnitude.

Of the other bright lines the *b* group and λ 500 were as broad but not so brilliant as Hβ; Hα and D were bright, while λ 466 first forced itself upon notice on this night. It has since remained a prominent member of the bright line series.

The seven dark spaces comprised more refrangible companions to Hα and Hβ, dark D_3, and dark spaces in the green less refrangible than *b*, λ 500, and Hβ. The remaining dark space intervened between λ 466 and Hγ. Their mean wave-lengths would be approximately λ 540, λ 510, λ 498, and λ 445. Of these, λ 510 and λ 498 were the most marked. There was also continuous spectrum in the region from D to λ 540, in which, however, no details could be made out with certainty.

On March 25 the star was just visible to the naked eye, being estimated as equal to *l*, or 32 of Cottam's charts, which is of magnitude 6. The bright lines seen were Hα, the line in the red about λ 640, D, *b*, λ 500, Hβ, λ 466; a wonderful spectrum for visually so small a star. The dark spaces were λ 540, λ 510, λ 498, which was between two and three times as broad as Hβ, and the dark space more refrangible than Hβ about λ 480, which, too, was very broad. The spectrum in its main details was not unlike that of March 21, except that it was less brilliant. Hα, however, and the bright space at λ 466 were quite as bright as on the former date.

On March 27 the star's magnitude had risen again to nearly 4, when first observed at 9.30, though at 10 P.M. it had sunk about half a magnitude. The bright lines seen were Hα, now the brightest of all; D, not so bright as on the 25th; *b*, only glimpsed; λ 500, well seen; Hβ, not very brilliant and seemingly diminished; λ 466, brighter and well seen; and Hγ. The dark companion to Hα was just visible, as was also dark D_3, and λ 498 and the companion to Hβ were seen.

On March 28 the star had again fallen to magnitude 6, and became even fainter during the course of the observations. The spectrum resembled that of the 25th, the luminosity having shifted to the red and orange. There was a very great increase in the brilliancy of the D line, and Hα was, too, very bright, equalling Hβ in intensity. The two faint lines, one on each side of Hα, first seen on March 21, were again visible. The *b* group was very faint and occasionally visible, but λ 500 almost equalled Hβ in brightness. Hβ, however, was less brilliant than on the 21st. The band λ 466 had increased in brightness, and Hγ was seen. The continuous spectrum in the green was still visible. The dark spaces λ 540, λ 510, and λ 498 were less marked.

On April 1 the magnitude of the star had again increased, being slightly above κ *Persei*, or 4·5 magnitude. Again, as on March 27, an increase in the magnitude of the star was accompanied by a diminution in the brilliancy of the D line; Hα was very bright, and its dark companion was seen. The *b* band was fainter, and was observed on this night for the last time. The band λ 466 was broader, and so, too, was the bright band still further to the violet, seen first on March 21. The dark spaces λ 510, λ 498, and λ 480 were present though not so distinct.

On April 16 the star was just on the limits of unaided vision,

being glimpsed a few times. It was estimated as less than 36 of Cottam's charts.

A great change had now taken place in the order of brilliancy of the lines, the order being λ 500, Hβ, λ 466, Hα, which was well seen at times, and D, seen only occasionally. Group *b* had disappeared, and the continuous spectrum in the green was of diminished lustre. The dark spaces λ 510, λ 498, and λ 480 were very prominent, partially, no doubt, due to contrast.

On April 19 *Nova* was invisible to the naked eye, and the observation was made through haze and fog. But λ 500 and Hβ seemed to be of equal intensity ; D was seen once or twice distinctly, Hα and λ 466 only glimpsed.

On April 21 the magnitude was equal to that of *l Persei*, or 6. The order of brightness was λ 500, Hβ, λ 466, which equalled Hβ at its least refrangible side and faded away towards the violet, D, which was faint, and Hα glimpsed once. The dark space λ 498 was very dark, and λ 480 less so. The continuous spectrum in the green had much brightened.

On April 23 the star rose in magnitude a little above *l*, and the order of brightness in the lines changed to Hα, Hβ, λ 466, λ 500, D. The continuous spectrum in the green still maintained its brilliancy.

On April 24 the star was observed in a hazy sky, but still seemed greater than *l*. The brightest line was λ 500, which was very broad. Then came Hβ, Hα, λ 466, which was brightest at its red side, was very broad, and faded away towards its violet side, and D, seen rarely. The dark spaces λ 510, λ 498, were seen.

On April 25, with the magnitude less than *l*, the order of brightness in the lines of April 16 was resumed, λ 500 being most prominent. The continuous spectrum was weak.

On April 26, with the same magnitude and with perfect seeing, the same order was maintained, except that Hγ was also seen. The D line was excessively faint. Although the continuous spectrum in the green was not very bright, it was present, and details were glimpsed in it beyond the power of the instrument to define. The dark space λ 498 was very pronounced, while λ 510 and λ 480 were well seen.

The chief conclusions to be derived from these details are as follows :

(1) When the star waned on March 25 and 28, the D line and the lines observed in the red and yellow end of the spectrum became bright. On March 27 and April 1, when the star became brighter, D and this part of the spectrum became less brilliant.

(2) The similarity in the order of the brightness of the lines on April 16, 21, 24, 25, 26, is noticeable. But on April 23, the star becoming brighter than on the 21st, the spectrum altered and became almost exactly like that of April 1 and March 27.

(3) The band at about λ 466, first seen on March 21, seemed to be growing relatively brighter and broader, until it became

quite conspicuous on April 16, and remained so, being brighter and sharper at its less refrangible end and diffuse in the opposite direction.

(4) Hα was very bright on March 12, 16, 21 ; was equal in intensity to Hβ on the 25th ; greater than it on the 27th, when it was the brightest of all the lines ; equal to it on the 28th ; greater than it on April 1 ; less on the 16th ; faint on the 19th and 21st ; greater yet again on the 23rd ; then less on the 24th, 25th, and 26th, on which last date it was very faint.

(5) The line at about λ 500 on March 21 was not as bright as Hβ ; on the 28th became equal to it, faded when the star rose in magnitude on April 1, became greater than it on the 16th, again equalled it on the 19th, became greater on the 21st, less on the 23rd, and the brightest of all the lines on the 24th, 25th, and 26th.

(6) The bright lines shown in this small instrument seem to be coincident with prominent chromospheric lines in the same region of the spectrum.

Stonyhurst College Observatory:
1901 *May* 1.

Further Observations of the New Star in Perseus, made at the Radcliffe Observatory, Oxford.

(*Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S., Radcliffe Observer.*)

This paper is a continuation of two others published in the last two numbers of the *Monthly Notices* (pp. 348 to 354 and 390 to 395). It contains the results of the observations of the magnitude and colour of *Nova Persei* made at the Radcliffe Observatory since the last meeting of the Society.

The estimates included in this paper have been chiefly made with telescopic aid, as encroaching twilight and the diminishing altitude of the *Nova* rendered naked-eye estimations increasingly difficult.

In this set of observations the number of comparison stars has been so limited that, to avoid the necessity of frequent reference to the previous papers, their names and index numbers have been set down in Table I. In cases where a telescope has been used an asterisk is affixed to the reference number.

Table II. contains the means of each observer's separate comparisons, and the general mean for each evening.

The accompanying diagram represents graphically the fluctuating changes in the brightness as evidenced by the Radcliffe observations, and is in continuation of a similar diagram printed on page 391 of the *Monthly Notices.* Upon examination of these

two diagrams it will be noticed that the interval between maximum and minimum brightness seems to have undergone a gradual lengthening, but that the range of variation (as far as the breaks in continuity permit us to form an estimate) appears to have changed but little. The greatest range of magnitude during the period under review occurred within an interval of two days only, viz. between April 21 and 23, 6·00 and 4·36 respectively.

The variations in the colour of the star have been noticed by all the observers, and the suggestion made in the last paper that the change in tint was closely related to fluctuations in brightness has been fully confirmed. In each case where the star has been observed near its minimum the colour has been described as increased in redness, and at maximum an orange tint has been usually noticed.

The difficulty of determining the magnitude of the *Nova* by means of comparisons with the neighbouring stars, all of which exhibit a very different quality of colour—which was remarked upon in the previous papers—has been emphasised during the present period by increasing twilight and the fact that but little opportunity is now afforded for observation owing to the rapidly decreasing altitude of the star after sunset.

TABLE I.

List of Stars used for Comparison with Nova Persei.

Ref. No.	Name of Star.	Harvard Photom. Mag.	Ref. No.	Name of Star.	Harvard Photom. Mag.
40	κ Persei	3·95	53	36 Persei	5·40
42	ν Persei	4·00	54	Arg. Z. +44°734	6·04
47	ψ Persei	4·24	55	Arg. Z. +44°648	6·47*
48	σ Persei	4·39	56	Arg. Z. +43°818	5·97
50	l Persei	4·84	57	Arg. Z. +42°754	8·0†
52	30 Persei	5·37	58	Arg. Z. +43°730	7·0†

TABLE II.

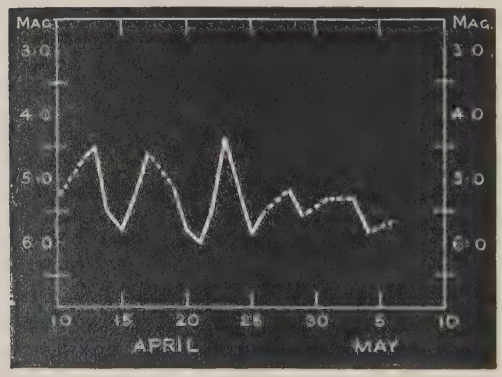
Means of Estimations of Magnitude of Nova Persei.

1901.	G.M.T. h m	Observer.	Reference Stars.	Mean Mags.	Adopted Magnitude for the Evening.
April 12	9 30	R.	42, 40, 50	4·70	4·67
	9 45	C.	50, 40, 42	4·63	
13	8 45	R.	42*, 40*, 50*	4·40	4·49
	8 50	W.	50*, 52*	4·75	
	10 0	C.	42, 40, 50	4·40	

* Revision of D.M. † D.M. mag. (Argelander.)

1901.	G.M.T. h m	Observer.	Reference Stars.	Mean Mags.	Adopted Magnitude for the Evening.
April 14	8 55	R.	53, 52, 54	5·50	5·50
	9 40	W.	52*	5·50	
15	9 0	R.	52, 54	5·65	5·82
	9 30	C.	52*, 50*	5·70	
	10 30	W.	52*, 57*	6·40	
16	9 0	R.	52*	5·40	5·32
	10 5	W.	50*, 52*	5·15	
	10 15	R.	52, 50	5·40	
17	7 30	R.	40*, 50*	4·55	4·62
	8 0	R.	50*	4·60	
	9 0	R.	40*, 50*	4·50	
	9 38	W.	50*	4·80	
19	8 23	W.	52*, 50*	5·20	5·13
	8 35	R.	50*, 52*	4·95	
	10 0	C.	52*, 50*	5·25	
20	7 45	R.	56*, 52*	6·00	5·83
	9 0	A.A.R.	52*, 55*	5·95	
	9 0	W.	52*, 55*	5·73	
	9 23	R.	54, 52, 53	5·73	
	9 30	C.	52*, 50*	5·65	
	10 50	C.	55*	6·00	
21	9 0	W.	52*, 55*	6·15	6·00
	9 25	R.	54, 53, 52	5·87	
22	8 23	W.	52*	5·60	5·47
	8 40	R.	52*, 56*	5·55	
	9 0	R.	52*, 56*, 54*	5·50	
	9 8	W.	52*	5·20	
23	9 17	R.	40, 42, 50	4·27	4·36
	9 20	W.	48*, 47*, 40*, 42*, 50*, 40*, 50*	4·47	
	9 20	W.	42, 40	4·50	
	9 38	C.	50*, 40*, 42*	4·27	
24	8 58	W.	52*	5·40	5·08
	9 8	C.	50*, 52*, 40*	4·90	

1901.	G.M.T. h m	Observer.	Reference Stars.	Mean Mags.	Adopted Magnitude for the Evening.
April 25	8 40	W.	54*, 53*, 52*, 55*	5·58	5·86
	9 15	C.	52*, 56*, 55*	6·13	
26	9 52	W.	53*, 52*	5·40	5·55
	10 45	C.	52*, 56*	5·70	
28	8 40	W.	52*, 50*	5·20	5·20
29	9 43	R.	52*, 56*, 50*	5·56	5·56
May 1	7 50	W.	50*, 40*	4·60	5·32
	8 0	R.	52*	5·50	
	8 5	W.	52*	5·50	
	8 37	C.	52*, 50*, 56*	5·37	5·34
	8 43	W.	52*, 50*	5·30	
4	8 53	W.	52*	5·60	5·83
	8 55	C.	52*, 58*, 54*, 56*	6·00	
	9 20	W.	52*, 55*	5·65	
	9 30	W.	52*, 55*	5·75	
6	9 5	R.	52*, 56*	5·70	5·70
	9 8	C.	52*, 56*	5·70	



Magnitudes of Nova Persei.

Observers' Notes on the Estimations of Magnitude.

1901.
Apr. 13. Sky misty, but much fainter stars well seen (W.).
Apr. 14. The Nova seemed fainter than 52 when seen earlier in the twilight (R.)
Apr. 15. With the $1\frac{3}{4}$-inch the Nova requires a different focus to surrounding stars. Sky very clear (W.).
Apr. 16. Occasionally Nova appeared equal to 52, but brightened up at times and excelled it; the image of Nova was, however, in the $1\frac{3}{4}$-inch telescope, larger in area than 52 (W.).
Apr. 17. Observed in strong twilight at 7.30 (R.). Sky misty over Perseus (W.).
Apr. 19. Sky is misty, but in the $1\frac{3}{4}$-inch the small stars between Nova and 50 are easily visible (W.). Difficult to estimate magnitude of Nova to-night, owing probably to slight atmospheric changes; or is it possibly due to rapid physical variations in the object? (R.).
Apr. 20. Differences in colour make the estimation of magnitude very difficult. $\frac{1}{2}$ wt. to 55 (W.).
Apr. 22. Some mist about at 8 23 (W.).
Apr. 23. Moonlight strong, but sky very clear (W.).
Apr. 24. Nova is at times visible to naked eye, but scintillating and vanishing; misty (W.). Thin cloud about; moonlight and twilight troublesome (C.).
Apr. 25. Some twilight and strong moonlight (W.). Estimations difficult; strong moonlight and twilight (C.).
Apr. 26. Nova very low.
Apr. 28. Twilight very strong.
Apr. 29. Nova very low in mist. Estimations difficult. $\frac{1}{2}$ wt. to 50.
May 1. Observed in very strong twilight at 7·50. By setting successively and as rapidly as possible on the three objects, with Mr. Robinson's assistance, I considered, by a single estimate, that, taking 10 steps between 40 and 50, Nova was 7 steps fainter than 40 and 3 steps brighter than 50. As the twilight waned the Nova exhibited a duller light in comparison with the other stars (W.). Twilight rather strong (R.).
May 3. Strong twilight (C.). In the 10-inch Nova required a different focus to the comparison stars (W.).
May 4. Strong twilight (C.). At 9.20 I can just see the stars of $7\frac{1}{2}$ magnitude which lie between Nova and 50 (W.).

Observers' Remarks on Colour of Nova and Comparison Stars.

1901.
Apr. 12. $10^h\ 15^m$. Nova, in Barclay 10-inch, power 90, red, but not very red (R.).
Apr. 13. $8^h\ 30^m$ to $10^h\ 15^m$. Nova, in 10-inch, power 90, red, but not strikingly so (C.). At 8.45 Nova, in Barclay "finder," red, but not very red. No. 40, yellow (R.). With the $1\frac{3}{4}$-inch, power 20, I noticed at 8.50 a reddish tinge in Nova, but the star was more yellow than on April 10 (W.).
Apr. 14. Nova reddish-yellow in $1\frac{3}{4}$-inch (W.).
Apr. 15. Nova is very red in the 10-inch: it has an orange centre with ruby fringes (R.).
Apr. 16. Nova, in 10-inch, power 45, is very red (R.). Nova to-night seems to be more red in the $1\frac{3}{4}$-inch than on Apr. 15 (W.). With 10-inch, power 90, Nova is more red than on Apr. 13 (C.).
Apr. 17. At $8^h\ 0^m$, centre of image of Nova yellow, with very red fringes, in 10-inch, power 45; but in "finder" orange-red only, less red than on Apr. 16 (R.). The Nova was found to be less red at $9^h\ 0^m$ than in the twilight (R.). Nova reddish-yellow in

1901.

Marlborough at 9.0 (W.). The Nova gradually became less red as it sank lower into the mist and cloud; the fringe had nearly disappeared at 10.30 (R.).

Apr. 19. Reddish-yellow in the $1\frac{3}{4}$-inch (W.). The star was very red in the 10-inch: its redness was maintained during the evening, 7.40 to 10.20 (R.).

Apr. 20. Nova very red in the 10-inch (R.). No. 55 is orange (W.). Nova red in Marlborough (C.).

Apr. 22. Image of Nova red in $1\frac{3}{4}$-inch (W.). Centre of Nova orange with red fringe in 10-inch; redness not so marked as on Apr. 20, certainly not so much as on Apr. 19 (R.).

Apr. 23. Nova not so red in $1\frac{3}{4}$-inch, but is a deep orange (W.). Nova less red in Marlborough telescope than on Apr. 20; to the naked eye, stars scintillating greatly (C.).

Apr. 24. The difference of colour of Nova and 52 is very marked. Nova is more red than on 23rd (W.). Nova more red in 10-inch than on 23rd (C.).

Apr. 25. With $1\frac{3}{4}$-inch Nova is dull reddish; compared with all other stars it appears a "flatter" colour. Much deeper colour and duller than No. 40 (W.). Nova is very red (C.).

Apr. 26. Nova is too low for definite colour observation in the $1\frac{3}{4}$-inch, but is certainly reddish and dull in contrast to No. 40 and other stars (W.).

Apr. 28. Nova reddish in $1\frac{3}{4}$-inch (W.).

Apr. 29. Nova very red to-night in 10-inch in contrast to the comparison stars (R.).

May 1. Nova yellowish-red. No. 40 bright yellowish. Nos. 52 and 50, white (W.).

May 3. Nova red in Barclay "finder" (C.). Nova reddish, in strong contrast to 52, in the 10-inch and "finder" (W.).

May 4. Nova very red in the 10-inch (C.).

May 6. Ruby fringes to image of Nova in 10-inch (R.). Very red in 10-inch (C.).

May 8. Caught a glimpse of the Nova in a break at 8.25 in the 10-inch telescope. Could not estimate magnitude, but from the absence of any marked colour I suspect the Nova to be near one of its maxima (R.).

The observers were: Dr. Rambaut, indicated by A.A.R.
Mr. Wickham, „ W.
Mr. Robinson, „ R.
Mr. McClellan, „ C.

Radcliffe Observatory, Oxford:
1901 *May* 9.

*Additional Note on the Position of Nova Persei, and a Comparison of Photographic Magnitudes of Neighbouring Stars with those of Fr. Hagen's Chart and Catalogue.** By F. A. Bellamy, F.R.Met.Soc.

In a former paper (*Monthly Notices*, lxi. p. 340) the position of the *Nova* and comparison stars were based upon the 31 stars in the Bonn A.G. Catalogue $+40°$ to $+50°$ reduced to epoch 1900·0. It will be seen on reference to Table I. (p. 341) that two stars 2972 and 2983 have rather large residuals in R.A., $+0^s·55$ and $+0^s·33$ respectively. These quantities being of the same sign and both the stars coming into the same sides of the equations, east and south halves of the plate, I thought the plate constants might be improved by omitting these two stars, and accordingly obtained these four equations (the computations of ξ' and η' of the Bonn stars, and the means of the Oxford measures being carried to the fourth decimal of the réseau intervals) :

	$x-\xi'$		$y-\eta'$
West half	$8·4362a + 13·7498b + c = +1·2725$	and	$+1·4199$
East half	$18·3311a + 13·7823b + c = +1·2671$		$+1·4723$
North half	$13·0493a + 18·6997b + c = +1·2444$		$+1·4426$
South half	$13·3658a + 9·1601b + c = +1·2936$		$+1·4476$

... (A)

When solved in the usual way the following constants for plate 1728 are obtained :

a	b	c	d	e	f
$-·00053$	$-·00518$	$+1·3482$	$+·00530$	$-·00035$	$+1·3800$

and the position of the *Nova* becomes

	h m s	° ′ ″	ξ	η
R.A. 1900·0	3 24 24·160	Dec. $+43$ 33 42·45	$-1·7920$	$-5·2585$

The declination of the *Nova* (*Monthly Notices*, lxi. p. 342) should have been given as $+43°\ 33'\ 42''·51$; an error ($0^{r.i.}·010$) was made in the very last step of the calculations ; the whole of the work has been re-examined, but no other error found. In the former paper the plate constants were obtained with the inclusion of these two stars ; for comparison, the equations, constants, and position of the *Nova* are here given.

* Georgetown College Observatory, Washington, 2nd Catalogue and Chart of *Nova Persei* stars.

		$x-\xi'$		$y-\eta'$.
West half	$8{\cdot}436a + 13{\cdot}749b + c = $	$+1{\cdot}2723$		$+1{\cdot}4199$
East half	$18{\cdot}010a + 12{\cdot}932b + c = $	$+1{\cdot}2740$	and	$+1{\cdot}4707$
North half	$13{\cdot}049a + 18{\cdot}699b + c = $	$+1{\cdot}2443$		$+1{\cdot}4427$
South half	$13{\cdot}648a + 8{\cdot}904b + c = $	$+1{\cdot}2969$		$+1{\cdot}4489$

... (B)

a	b	c	d	e	f
·00031	−·00539	+1·3491	+·00528	−·00031	+1·3797

	h m s	° ′ ″	ξ	η
Nova 1900·0	3 24 24·12	+43 33 42·3	−1·794	−5·259

The differences, $x-\xi'$ and $y-\eta'$, were entered to the third decimal place for each star, the means of groups being carried to the fourth place. As it might be considered by some that this was insufficiently accurate, I have reformed the equations, entering the x, y, $x-\xi'$, and $y-\eta'$ quantities to the fourth place throughout, and they become

West half	$8{\cdot}4362a + 13{\cdot}7498b + c = $	$+1{\cdot}2725$		$+1{\cdot}4199$
East half	$18{\cdot}0100a + 12{\cdot}9323b + c = $	$+1{\cdot}2740$	and	$+1{\cdot}4706$
North half	$13{\cdot}0493a + 18{\cdot}6997b + c = $	$+1{\cdot}2444$		$+1{\cdot}4426$
South half	$13{\cdot}6477a + 8{\cdot}9039b + c = $	$+1{\cdot}2970$		$+1\ 4490$

... (C)

The constants found from these are almost identical with (B).

a	b	c	d	e	f
−·00033	−·00539	+1·3495	+·00527	−·00033	+1·3800

	h m s	° ′ ″	ξ	η
Nova 1900·0	3 24 24·122	+43 33 42·51	−1·7939	−5·2583

A comparison of (B) and (C) seems to show that no sensible increase in accuracy is obtained by extending the calculations to the fourth place of decimals, *which more than doubles the expenditure of time*. A considerably greater change might be made in the constants by the inclusion or omission of a single comparison star (*étoile de repère*) as in (A). This evidence may be of interest to those engaged in the Astrographic Survey.

Passing on to the consideration of the measures of another plate 1732 taken, as before, with the Astrographic Telescope on 1901 March 5 between Oxford sidereal time $6^h\ 24^m\ 40^s$ and $6^h\ 32^m\ 10^s$ with exposures of 1^m, 1^m, 1^m, I will state briefly that each exposure of the thirty-three Bonn stars was measured in D- and R-positions by myself, and the means of these six bisections were taken to four places of decimals ; they were then compared with the Oxford standard coordinates ξ' and η' given in Table I. (*Monthly Notices*, lxi. p. 341), the equations formed and solved in the usual way—the two stars 2972 and 2983 being omitted—and the separate stars were corrected for these constants.

In the case of the *Nova*, the diameter of the photographic

image being 30″, three independent bisections of each exposure were made in D- and R-positions of the plate in the measuring instrument, and, similarly, a pair of contact measures were obtained; the means of the eighteen bisections and thirty-six contacts in x and y are

x		y	
Bisections. r.i.	Contacts. r.i.	Bisections. r.i.	Contacts. r.i.
12·7493	12·7491	9·2708	9·2707

With a view to ascertain if there were any personality in the measurement of large images, *Nova* on this plate was measured in a similar manner by Messrs. H. F. Mullis, B. Gray, and E. A. Gray, with these results:

	x		y	
	Bisections.	Contacts.	Bisections.	Contacts.
H.F.M.	12·7487	12·7499	9·2708	9·2706
B.G.	12·7495	12·7498	9·2692	9·2707
E.A.G.	12·7479	12·7489	9·2698	9·2717

The personality, if any, is very small.

The means in x are 12·7489 for bisections and 12·7494 for contacts, and in y 9·2702 for bisections and 12·2709 for contacts. The equations for plate 1732 are:

$$\left.\begin{array}{ll} \text{West half} & 8{\cdot}2366a + 13{\cdot}8810b + c = +1{\cdot}0728 \\ \text{East half} & 18{\cdot}1309a + 13{\cdot}8117b + c = +1{\cdot}0689 \\ \text{North half} & 12{\cdot}8994a + 18{\cdot}7832b + c = +1{\cdot}0943 \\ \text{South half} & 13{\cdot}1193a + 9{\cdot}2411b + c = +1{\cdot}0491 \end{array}\right\} \quad x-\xi' \quad \text{and} \quad \left\{\begin{array}{l} +1{\cdot}5512 \\ +1{\cdot}5017 \\ +1{\cdot}5262 \\ +1{\cdot}5283 \end{array}\right. \quad y-\eta'$$

The constants obtained are

a	b	c	d	e	
−·00036	+·00473	+1·0101	−·00500	−·00033	+1·5969

Taking the means of the measures, 216, each in x and y, made by the four measurers, and correcting them for these constants, we get $11^{\text{r.i.}}6998$ for ξ' and $7^{\text{r.i.}}7404$ for η', which, converted into R.A. and Dec., give positions for *Nova* for 1900·0:

	h	m	s		°	′	″		
R.A.	3	24	24·116	Dec.	+43	33	41·79	ξ −1·3002	η −5·2596

By using the same constants for correcting the mean measures of the Bonn stars this table is formed:

TABLE I.

Bonn. A.G.C.	Residuals. x r.i.	x "	y r.i.	y "	Bonn. A.G.C.	Residuals. x r.i.	x "	y r.i.	y "
2883	·0000	·00	−·0001	−·03	2970	−·0004	−·12	+·0003	+·09
2892	+·0003	+·09	+·0005	+·15	2971	+·0005	+·15	+·0009	+·27
2895	+·0007	+·21	+·0009	+·27	2973	+·0005	+·15	+·0010	+·30
2898	+·0012	+·36	−·0007	−·21	2979	−·0015	−·45	−·0002	−·06
2899	+·0023	+·69	+·0003	+·09	2982	+·0008	+·24	+·0003	+·09
2906	−·0007	−·21	−·0007	−·21	2986	−·0002	−·06	−·0005	−·15
2907	−·0002	−·06	−·0002	−·06	2996	−·0008	−·24	−·0006	−·18
2911	+·0006	+·18	+·0005	+·15	3004	−·0010	−·30	−·0002	−·06
2913	−·0004	−·12	+·0006	+·18	3008	+·0012	+·36	+·0002	+·06
2919	−·0015	−·45	·0000	·00	3009	+·0017	+·51	+·0009	+·27
2944	−·0004	−·12	·0000	·00	3016	−·0015	−·45	·0000	·00
2948	+·0010	+·30	+·0022	+·66	3021	−·0009	−·27	−·0010	−·30
2953	−·0008	−·24	−·0008	−·24	3034	−·0004	−·12	−·0003	−·09
2956	+·0008	+·24	−·0019	−·57	2972	−·0011	−·33	−·0011	−·33
2964	−·0023	−·69	−·0010	−·30	2983	+·0010	+·30	+·0008	+·24
2968	+·0024	+·72	−·0005	−·15					

This table is instructive, as the quantities given are really the differences in the measured positions of the same stars on two plates, and contain all errors in bisections, determination of plate constants, and in the reductions. It may be noticed that the maximum difference in x is less than three quarters of a second of arc, and that there are not more than four stars with differences exceeding half of a second of arc; in the y coordinate there are two instances above a third of a second of arc, but the differences are generally smaller; the mean difference in x without regard to sign is $0''\cdot279$, and in y $0''\cdot188$. With regard to the last two stars, which have not been included in the reductions, the rather large differences as indicated in the former paper disappear, and those differences cannot be in the measures, but must be either in the Bonn positions or caused by proper motions.

For convenience the places of the *Nova* may be repeated here.

	R.A. 1900·0 h m s	Decl. 1900·0 ° ′ ″
1901 Feb. 25	3 24 24·160	+43 33 42·51
1901 March 5	24·116	41·79

Comparison of Photographic and Visual Magnitudes.

In the former paper no attempt was made to determine the magnitudes of the 159 stars on Argelander or any definite scale, the measured diameter only being given. Since that paper was written Fr. Hagen, of Georgetown College Observatory, has published a second chart and catalogue of faint stars for com-

parison with *Nova* if it should fall below the 7th to the 13th magnitude. In order to ascertain whether all the stars in Fr. Hagen's second catalogue are on plate 1728, and also to gain some information of the estimated as compared with the photographic magnitude, expressed in thousandths of a réseau interval (300″), I have compared the stars in his catalogue with those places obtained from plate 1728, and give the comparison in Table II. Stars No. 1 to 10, 12 to 16, 19, 23, 25–28, 30, 33, 35, 39, 40 are outside the area included in the former Table II. (p. 343).

In making the comparison the position of *Nova* has been taken as $3^h\ 24^m\ 24^s\ +43°\ 33'\ 42''$.

Table II.

B.D.	Hagen.	Oxford.	H.	O.	Difference. O−H.		B.D.	Hagen.	Oxford.	H.	O.	Difference. O−H.	
No.	No.	No.	Mag.	Mag.	R.A.	Dec.	No.	No.	No.	Mag.	Mag.	R.A.	Dec.
°					s	″	°					s	″
43·734	36	5	9·6	12	− ·5	+43		75	80*a*	12·5	3	− ·7	+10
	43	23	10·2	12	− ·4	+ 6		76	80*b*	12·6	4	− ·3	+14
	45	27	10·4	11	+ ·3	+ 0		74	81*a*	12·4	7	− ·8	+ 2
	46	30	10·7	11	− ·4	+ 1	43·741	29	86	9·2	14	+2·2	+23
	52	31	11·2	8	+ ·3	+ 3		50	89	11·1	7	+ ·2	− 5
	51	32*b*	11·2	11	·0	+ 5		71?	89*a*	12·3	5	+5·2	+ 3
43·735	34	37	9·6	12	+ ·2	+ 2		66	90	11·9	6	− ·1	− 6
	61	39	11·7	8	+ ·1	+ 5		68	91	12·1	8	−1·1	− 3
	67	40*a*	12·0	5	·0	0		42	92	10·1	12	+ ·6	− 3
	60	42	11·7	9	+ ·3	−18		55	93*a*	11·4	7	+ ·7	+ 9
	63	44	11·9	7	− ·3	− 2	43·743	32	94	9·4	14	+ ·2	− 8
43·737	38	48	9·8	11	+ ·3	− 7		65	93*b*	11·9	5	+ ·6	− 2
	58	50	11·6	7	+ ·1	− 2	43·744	11	98	8·4	24	+ ·9	+ 1
	64	51	11·9	7	+ ·1	+12		47	101	10·7	10	− ·4	−16
	53	55*a*	11·3	9	+ ·7	−10		48	103	10·8	10	·0	+16
43·738	37	57	9·7	9	+ ·6	−16		72	107*a*	12·4	3	−1·1	−13
	49	59	11·0	10	·0	+ 1	43·746	18	108	8·8	20	− ·3	+23
43·739	20	61	8·9	23	+ ·2	+ 1		70	109	12·1	7	·0	− 1
	54	62	11·4	8	+ ·6	− 1		59	111	11·6	8	+ ·4	+ 3
	56	64	11·4	10	−1·6	−23	43·748	24	127	9·0	23	+ ·2	− 4
	57	67*a*	11·6	8	− ·7	+72	43·749	21	132	9·0	15	+ ·2	− 1
43·740	17	68	8·8	21	+ ·6	− 2	43·750	41	137	10·0	14	+ ·2	−18
	44	71	10·3	11	+ ·2	+14	43·751	22	142	9 0	22	− ·3	− 3
	62	73	11·8	8	− ·4	−68	43·752	31	148	9·3	19	+ ·3	+25

Notes.

Hagen No. 29. The difference is rather large; there is a very faint star 4^s preceding, and 45″ north.

No. 51. Only the second exposure measured; the third exposure falls on the réseau line and is invisible.
Nos. 57, 62. No other stars are visible on the plate that would agree more closely.
No. 64. The sign of $\Delta\alpha$ should be *minus*; the chart is correct. A notification of this I have already communicated to the *Astronomische Nachrichten*.
No. 71. Extremely faint, just a trace of the second image; possibly Oxford 89α is another star, the other being too faint.
No. 72. Extremely faint, no trace of the second image.
No. 73. If the third and longest exposure is on the plate its place would be on a réseau line; second exposure invisible.
No. 77. Can find nothing to agree with this.
Nos. 74, 75. The second images barely visible, but were not measured; mag. of No. 74 not measured, very diffused image, not like a star; the latter is within the discoloured part of the plate caused by the brightness of *Nova*.
Nos. 69, 78, 79, 80, 81. These are too close to the *Nova* to be seen on the plate, and probably too faint.

The positions of faint stars within 3′ of *Nova* have been determined with the 36-in. telescope at the Lick Observatory (*Publications of the Astron. Soc. of the Pacific*, vol. xiii. p. 65).

As several stars could not be identified with certainty or were not previously measured on the plate, it was examined under the most favourable conditions as to light, and by knowing the positions in which to look for very faint stars. The result of this scrutiny is that every instance of doubt has been removed, except in the case of Hagen's Nos. 71, 73, 77. The identification has not been at all easy owing to other stars being within reasonably close agreement for approximate positions. Father Hagen says his places may be in error by 1^s in R.A. and 36″ in Dec. After some months' experience in comparing Oxford astrographic plates with B.D. stars, the magnitude of the differences, and the frequent uncertainty in deciding which amongst the numerous photographed stars to compare it with, one comes to the conclusion that we are getting too many stars to deal with unless accurate positions are determined; otherwise erroneous identifications will be made.

Nearly all the stars omitted before and now given in this paper (Table III.) are barely visible on the plate, and would easily be overlooked for the reasons just mentioned. For the astrographic work the measurers are generally instructed to pass by and not spend time over any very faint objects which do not show the second image obviously; we are seldom troubled with an insufficiency of stars on our plates, and it seems the better plan to omit doubtful images of stars (? spots in the film) rather than record a large number of spurious places of stars and create perplexities for the future.

The positions of the additional stars with the catalogue number are as below (Table III.).

Table III.

Hagen No.	Ref. No.	Oxford Measured Mag.	ξ' 1900·0.	η' 1900·0.	Deduced. R.A. 1900·0. h m s	N. Dec. 1900. ° ′ ″
	16 *a*	6	7·9564	5·6120	3 22 41·22	43 22 58·4
	20 *a*	5	8·2234	5·9673	22 48·50	24 46·9
	32 *a*	5	8·7893	5·7607	23 4·11	23 44·6
51	32 *b*	11	8·9069	8·3660	23 6·95	36 46·5
67	40 *a*	5	9·5845	6·0859	23 25·95	25 23·5
53	55 *a*	9	10·5353	10·8429	23 51·74	47 55·5
57	67 *a*	8	11·4580	10·2804	24 17·29	46 23·6
75	80 *a*	3	12·2855	5·3735	24 40·34	21 52·0
76	80 *b*	4	12·4079	5·5451	24 43·71	22 43·5
74	81 *a*	7	12·4265	8·0682	24 44·16	35 20·4
	84 *a*	6	12·6477	11·8462	24 50·15	54 13·8
71?	89 *a*	5	12·8203	6·1291	24 55·16	25 38·8
55	93 *a*	7	13·0268	4·8882	25 0·74	19 26·6
65	93 *b*	5	13·2379	8·6942	25 6·57	38 28·3
	96 *a*	7	13·3804	8·4747	25 10·51	37 22·4
72	107 *a*	3	14·1583	5·6918	25 31·88	23 47·4

As the ordinary photographic plate is not adapted for orange or red coloured stars, it is reasonable to suppose that they would not be on plate 1728, at least if fainter than about the tenth magnitude; this would not tend to agreement in visual and photographic magnitudes.

Grouping the magnitudes in Table II., the following results are obtained for this plate:

Table IV.

Limit of Magnitude.	Hagen Mean Mag.	Oxford. Plate 1728.	No. of Stars.
6·5 – 6·9	6·7	33	2
7·0 – 7·4	7·2	36	1
7·5 – 7·9	7·7	32	3
8·0 – 8·4	8·4	25	4
8·5 – 8·9	8·8	21	12
9·0 – 9·4	9·1	18	18
9·5 – 9·9	9·7	11	4
10·0 – 10·4	10·2	12	5
10·5 – 10·9	10·7	10·3	3
11·0 – 11·4	11·2	8·6	7
11·5 – 11·9	11·7	7·7	11
12·0 – 12·4	12·2	5·6	5
12·5 – 12·9	12·6	3·5	2

From which may be derived this *approximate* scale for interpreting the photographic magnitudes given for plate 1728.

Mag. ...	... 7·0	7·5	8·0	8·5	9·0	9·5	10·0	10·5	11·2	11·9	12·2	12·6	13·0
Oxford diameter	35	33	29	24	20	15	13	11	9	7	6	5	4

Estimations of Magnitude.

March 25, 7h. This evening being brilliantly fine, I watched for and noted the time of the first appearance to the eye of some stars of similar magnitude near the *Nova*. At 7h 24m G.M.T. saw ν (3·9 mag.); 7h 28m ι appeared (4·2); 7h 31m Hagen No. 26 and 27 (5·3 and 5·1) were seen, probably as combined light; 7h 32m *Nova* was seen; l (5·1) appeared some minutes before *Nova*. At 8h *Nova* estimated as $\frac{1}{2}$ magnitude fainter than l. Difference in colour would affect such observations.

March 26, 9h. Sky very clear, moonlight; *Nova* = ι and σ *Persei* and $\frac{1}{2}$ magnitude fainter than κ.

April 10. *Nova* intensely red (crimson).

University Observatory, Oxford:
1901 *May* 9.

Further Observations of the New Star in Perseus (3).

By A. Stanley Williams.

The following paper contains a further list of observations of the brightness of *Nova Persei* in the same form as the last one. As, however, the faintness of the star soon rendered greater optical aid necessary, a column has been added to indicate the instrument used. In this Op. signifies opera glass, $\frac{3}{4}$ in. the $\frac{3}{4}$-inch finder attached to a $2\frac{3}{4}$-inch refractor, and $2\frac{3}{4}$ in. the latter instrument itself, with a power of 35. The observations were much hindered by cloud in the early part of April, but the latter half of the month fortunately proved very fine.

Date. 1901.	Greenwich M.T. h m	Observations.	Mag.	Inst.
Apr. 12	9 35	κ Persei + 5, 32 Per. −1, 36 Per. − 5	4·7	Op.
13	10 20	κ Per. + 2, 32 Per. −6, 36 Per. − 8	4·35	,,
14	8 30	κ Per. +12, 32 Per. +5, 36 Per. + 1	5·35	,,
	10 10	32 Per. + 4, 36 Per. −2	5·2	,,
15	9 35	32 Per. + 6, 36 Per. +1	5·45	,,
	10 5	36 Per. + 2, 30 Per. +1	5·55	,,
17	9 45	32 Per. + 2, 36 Per. −3	5·05	,,
19	9 45	32 Per. + 5, 30 Per. −1, =36 Per.	5·35	,,
20	9 40	32 Per. + 6, 30 Per. +1, =36 Per.	5·45	,,
21	8 50	36 Per. + 3, 30 Per. +2, = a	5·9	,,
		36 Per. + 7, = a	6·3	$\frac{3}{4}$ in.
		36 Per. + 8, a+2 (see note)	6·25	$2\frac{3}{4}$ in.
	10 0	36 Per. + 2, 30 Per. +2, a−2	5·8	Op.

Date. 1901.	Greenwich M.T. h m	Observations.	Mag.	Inst.
Apr. 22	9 15	36 Per. + 5, a − 1	6·15	$\frac{3}{4}$ in.
		36 Per. + 7, a − 3	6·15	$2\frac{3}{4}$ in.
	9 40	36 Per. + 1, a − 3	5·85	Op.
	10 30	= 36 Per. 30 Per. −1	5·35	„
23	8 38	κ Per. + 1, ν Per. +5	4·3	„
	8 48	κ Per. + 2, ν Per. +6, 32 Per. −10	4·2	„
	8 55	κ Per. + 1, ν Per. +5, 32 Per. − 8	4·2	„
	9 9	κ Per. + 1, ν Per. +4	4·2	„
	9 36	κ Per. + 1, ν Per. +4, 32 Per. − 8	4·15	„
	10 6	κ Per. + 1, ν Per. +6	4·3	„
	10 28	κ Per. + 1, ν Per. +4	4·2	„
24	9 15	36 Per. + 7, a − 7	5·95	$2\frac{3}{4}$ in.
	9 25	36 Per. + 6, a − 4	6·05	$\frac{3}{4}$ in.
25	9 17	36 Per. + 7, a − 2·7	6·2	$2\frac{3}{4}$ in.
26	9 8	36 Per. + 8, a − 3	6·2	„
28	9 0	= 36 Per. a − 10	5·45	„
May 2	8 50	About 1 mag. brighter than 36 Per.	4·4	± „
3	8 45	36 Per. + 1·5, a − 9·5	5·55	„
4	8 45	36 Per. + 9, a − 2	6·3	„
5	8 36	36 Per. + 8, a − 3	6·2	„
6	8 35	36 Per. + 10, a − 1	6·4	„

Notes.

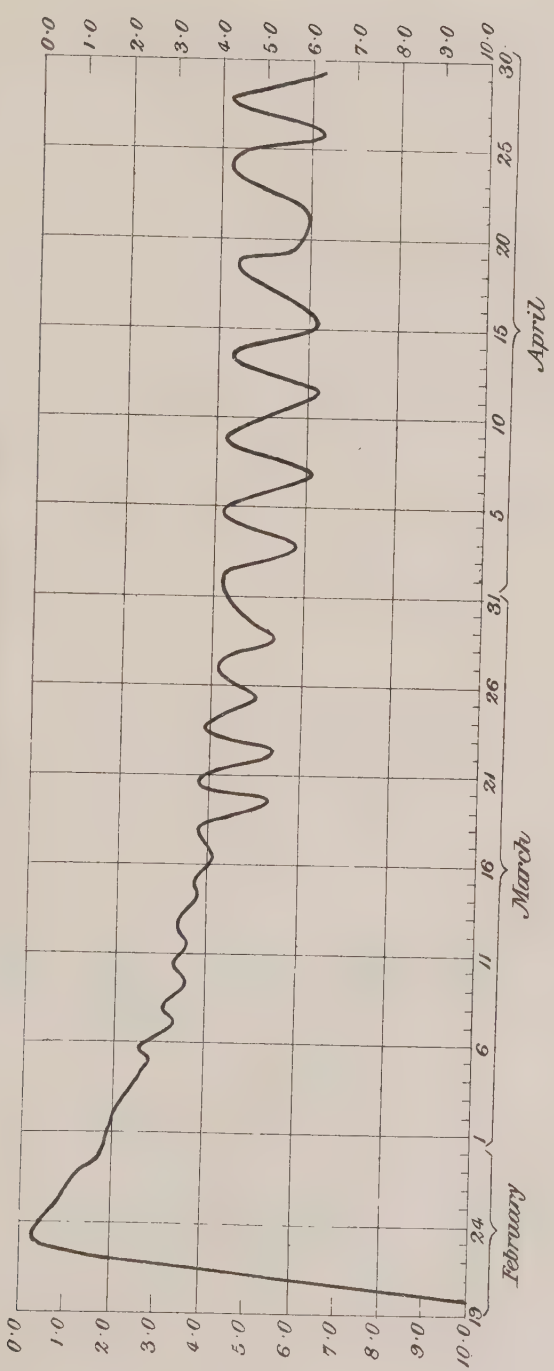
April 12, a little hazy but no cloud. April 13, slight mistiness may just possibly have affected the estimates. April 14, very clear. April 15, very clear. April 17, hazy. April 19, a little hazy towards horizon, otherwise clear. April 20, very clear, *Nova* just steadily visible in opera glass. April 21, exceedingly clear. A note to the last estimate made the same night runs, "I feel *sure* this should be $a-2$," instead of $a+2$. April 22, very clear. April 23, very clear except at commencement and end of the series, when slightly hazy. April 24, hazy. April 25, clear. The observations as given are the means of three separate sets of comparisons between $9^h\ 10^m$ and $9^h\ 25^m$. April 26, very clear, but some cloud about. Three sets of comparisons between $9^h\ 0^m$ and $9^h\ 18^m$. April 28, clear night. Good observations from many repeated comparisons. May 2, hazy and sky very bright, so impossible to make a satisfactory estimate. The star was invisible in the opera glass, and owing to a counterpoise coming in contact with one of the legs of the stand, it could not be followed long enough to make comparison with brighter stars, such as κ and ν *Persei*. The brightness of the *Nova* on this night is, however, certain. It was incomparably brighter than the star a, and a whole magnitude brighter than 36 *Persei*. May 3, very clear. Mean of two sets of comparisons between $8^h\ 40^m$ and $8^h\ 50^m$. May 4, hazy, but observations fairly satisfactory. May 5, clear. Mean of four sets of comparisons between $8^h\ 30^m$ and $8^h\ 43^m$. May 6, very clear. Mean of three sets of comparisons between $8^h\ 30^m$ and $8^h\ 41^m$.

The comparison star designated *a* in the above observations is the 6·5 magnitude star B.D.+44°·734. It will be seen that there have been some further remarkable fluctuations in the brightness of *Nova Persei*, and of a type apparently differing somewhat from that of the changes which occurred in March. In that month the normal condition of the star as seen here seems to have been bright, with well marked and rather accentuated minima at intervals of three or four days. But in April-May the star has appeared normally faint, with at intervals well marked and accentuated maxima. The observations show that the star was unusually bright on April 13, 17, 23, and 28, and on May 2. The maxima of April 23 and May 2 are specially remarkable. On the former of these two dates the star appeared nearly two whole magnitudes brighter than it had been the previous night. So striking was the change that observations were repeated at intervals to see if the star was still varying. There was no certain change, however, in the interval of $1^h\ 50^m$ over which the observations extended, although by the following night the star had sunk again to 6·0 mag. On May 2 the rise seems to have been nearly as great as it was on April 23. The conditions were very unfavourable on this night, but I feel sure, notwithstanding, that the *Nova* was a full magnitude brighter than 36 *Persei*, or about 4·4 mag. It is easy to trace a rough periodicity in the above dates of maxima, but observations from the other hemisphere seem necessary for the satisfactory discussion of such a question.

The observations made with different instruments at the same time on one or two nights, April 21 in particular, present some rather curious differences, and it would seem that the opera-glass is not a suitable instrument for noting slight differences of brightness in stars nearly at the limit of visibility. Under such conditions the value of a step seems to be markedly greater with the opera-glass than it is with either the $\frac{3}{4}$-inch or $2\frac{3}{4}$-inch telescopes, and the estimates correspondingly rougher.

The following observations of the colour of the *Nova* were made with p. 75 on the $2\frac{3}{4}$-inch refractor. April 12, fairly deep reddish at $9^h\ 35^m$, but at $9^h\ 55^m$ white without any decided red tinge. April 15, flaming red, a very bright intense red colour. April 19, very intensely red (crimson). April 20, crimson, pretty deep in flashes, though not quite as intense as on the previous night. April 21, reddish, not at all deep or marked. April 22, reddish, not at all deep or marked. April 23, white or slightly yellowish, with pale reddish flashes. The red colour not at all marked and not *nearly* so deep as it was a few nights earlier. At times, indeed, there seemed to be distinct bluish flashes. April 24, very deep intense red. April 26, very red.

The almost complete absence of red colour on April 23 is rather remarkable, since the star was then at a maximum of brightness. On April 24 the *Nova* had fallen to the 6th mag. and


Light Curve of Nova Persei 1901.
0·0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
19
24
1
6
11
16
21
26
31
5
10
15
20
25
30
February
March
April

had again become intensely red. This would seem to indicate a connection between the variations of brightness and colour. However, it should be noted that both on April 21 and 22, when the star was faint, the red colour was not at all deep or marked.

Hove: 1900 *May* 8.

Light Curve of Nova Persei, 1901. By Laurence Child.

(*Communicated by Rev. Edmund Ledger.*)

The curve (plate 13) is the mean of 527 observations. I have obtained the data from observations made by Miss Orr and myself, together with those published in the *Ast. Nach.* and the *Bulletin de la Soc. de France.* The only day without observations is April 24, so that the magnitude of the maximum on that day is only surmised.

The dates of maxima and minima are as follows :—

Minima.		Maxima.	
March 19		March 20	
22	Very red	23	
25	Red	27	Orange
28	Red	April 1	Orange
April 2	Red	4	
7	Red	9	
11	Red	14	
15	Red	19	Orange
21	Red	24	
25	Very red	27	

Vernham, Merton Hall Road, Wimbledon.

Further Observations of Nova Persei. By M. C. Sharp.

The following list of estimated magnitudes is rather short, cloudy weather and pressure of other occupations having somewhat curtailed opportunities :

April 12	4·7	April 20 ($10^h\ 40^m$)	5·6
,, 13	4·4	,, 21	5·7
,, 14	5·3	,, 22 ($8^h\ 48^m$) ...	5·5
,, 15	5·6	,, ,, ($10^h\ 15^m$)	5·2
,, 18	4·1	,, 25	6·0
,, 19	5·4	May 3	5·8
,, 20 ($9^h\ 45^m$) ...	5·8	,, 4	6·6(?)

At the time of the second observation on April 20 the sky was rather clearer than at the earlier hour. On April 22 at 8.48 there was still some twilight, and this probably accounts partly for the apparent rise in magnitude. On May 4 the moonlight and haze obliterated the fainter stars in the binocular. D.M. 46°·760, which was used as a comparison star, is rated at 6·5 in the Durchmusterung, but certainly appeared brighter as judged by other stars near which were visible.

1901 *May* 9.

The Green Flash at Sunset. By J. Franklin-Adams.

A series of notes as to the green flash at sunset suggests that time spent upon careful observation of this phenomenon would be amply repaid; not much more seems to be known about it than the fact that at a particular moment—when the Sun's image is disappearing below the sea horizon—a flash of brilliant green is in a clear sky visible to the naked eye during a fraction of a second.

With a little optical help—that, for instance, of a Zeiss prismatic field glass—it will be found that the green display continues for nearly three seconds.

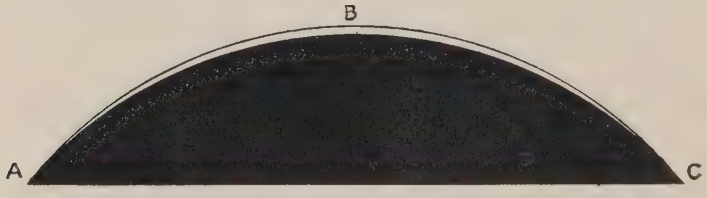


Fig. 1.

The object of this communication is to give some details of an observation made here on April 19 of this year. The Sun set in a manner exceptionally favourable for observing the green flash: sky quite clear, and without any trace of haze or mist even near the horizon.

The green flash first appeared in the shape of a single brilliant emerald bead, then another, and a third, and in about a second a full display of Baily's beads had developed in much the same way as in a solar eclipse, but to a much smaller extent and for a much shorter time. The first bead I noticed was towards the S. point of the horizon, the others close towards the N., until my eye reached the fourth, when I found a string of about seven beads. My idea is that they would form at either end of the segment D E F, fig. 2, and join on the middle of the row, but my

surprise at the sight of Baily's beads without an eclipse was so great that I did not notice this point.

I beg to submit that it would be interesting if careful observers by the seaside—or at sea—during the summer would send any results to the secretaries or to myself. Especially would I ask for sunrise observations of the flash. Astronomers after a night's work seldom sit up until sunrise, but on board ship it is easy to arrange to be called just before sunrise.

It has been suggested to me that if, as has been more or less accepted hitherto, Baily's beads owe their origin to the mountains and valleys in the Moon, the phenomenon here described may have been caused by the waves of the sea taking the place of the mountains of the Moon. The evening was calm, and the waves not large enough to make this possible unless images of the waves were thrown up by a mirage state of the atmosphere. I have twice seen from a steamer's deck mirage images of the sea waves, about a mile distant, thrown up sufficiently to entirely hide the horizon.

Fig. 2.

I add some further details and some diagrams. Time occupied in fully developing about one and a half seconds, and the same in disappearing. Time was estimated by counting aloud and not by chronograph. Length of bead display in measure of Sun's diameter about one-twelfth—this is very uncertain. Colour of beads emerald green with a slightly bluish tinge. Fig. 1 shows a Moon of 32′ eclipsing a Sun of 31′. Fig. 2 shows the extreme diametrical measure of the crescent A B C, fig. 1, being occulted by the straight line of the sea horizon. Fig. 3 shows an estimate of the proportionate size of the beads, drawn for the sake of clearness on a scale about four times that of figs. 1 and 2.

Fig. 3.

Endeavour will be made here during the summer, when our long twilight stops stellar photography, to photograph some flashes through colour screens or filters and to take chronographic measurements of their duration.

Machrihanish, Argyllshire : 1901 *April.*

M M

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1900.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, aperture 28 inches, focal length 28 feet. The power generally used was 670, but when the definition permitted a power of 1120 was employed for observing very close pairs. A blue glass shade was employed to diminish the light and irradiation when bright stars were observed. The observations were made in variously coloured fields or in a dark field with illuminated wires. The initials in the last column are those of the observers, viz. :—

B. Mr. Bryant L. Mr. Lewis W.B. Mr. Bowyer

The observations of *Capella* have already appeared in the *Notices* of the Society for 1900 June and December, and hence are not included in the present paper.

Micrometric Observations of Double Stars.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle °	Distance. ″	No. of Meas.	Mags.		Epoch 1900.	Obs.
Σ 3062 ...	0 1	32 10	341·8	1·50	1	6·9	8·0	·066	B.
β 253 ...	0 5	32 2	49·7	0·43	1	8·3	8·4	·066	B.
Krueger 1 ...	0 6	32 43	188·7	2·01	1	9·1	9·2	·066	B.
β 1026 ...	0 7	36 56	330·9	0·37	1	8·1	8·9	·066	B.
OΣ 2 A.B. ...	0 8	63 35	38·2	0·52	2	6·5	8·0	·033	B.
			37·3	0·49	1	...		·041	L.
OΣ 4 ...	0 10	54 4	100·6	0·23	2	7·4	8·1	·948	B.
β 1027 ...	0 10	69 3	180·1	1·40	1	7·7	10·5	·041	L.
			184·1	1·44	1	...		·747	W.B.
β 1093 ...	0 15	73 36	59·1	0·31	2	7·3	8·2	·493	B.
OΣ 12(λ Cassiopeiæ)...	0 26	36 8	145·0	0·37	1	5·0	6·0	·066	B.
OΣ 15 ...	0 30	41 32	320·5	0·17	1	7·5	8·5	·964	B.
β 257 ...	0 35	43 17	234·5	0·49	1	8·1	8·8	·964	B.
β 865 ...	0 38	47 20	195·9	1·11	1	8·3	8·8	·964	B.
β 866 ...	0 40	47 20	70·8	1·54	1	9·1	9·1	·964	B.
β 495 ...	0 42	71 52	225·6	0·57	2	7·6	7·7	·033	B.
β 232 ...	0 48	39 55	320·1	0·44	1	8·0	8·1	·964	B

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
OΣ 20 ...	0 49	71 22	327·9	0·33	1	5·9 7·0	·022	B.
			314·1	0·48	2	...	·931	W.B.
Σ 73 (36 Andromedæ)	0 49	66 55	19·1	0·90	4	6·2 6·8	·853	W.B.
OΣ 21 ...	0 57	43 10	29·5	0·97	1	6·9 8·2	·964	B.
β 303 ...	1 4	66 44	286·1	0·64	2	7·2 7·5	·033	B.
			291·5	0·62	1	...	·991	W.B.
β 4 ...	1 18	79 9	65·7	0·33	1	7·8 8·8	·022	B.
β 1164 ...	1 22	85 10	168·8	0·33	1	6·7 7·0	·022	B.
β 506 (η Piscium)	1 26	75 10	14·5	1·16	1	4·0 10·5	·047	L.
Σ 138 ...	1 30	82 52	220·7	1·45	1	7·3 7·3	·044	B.
			217·6	1·43	1	...	·047	L.
Σ 155 ...	1 35	8 34	328·9	4·53	1	7·5 7·9	·047	L.
β 509 ...	1 38	80 56	268·7	0·69	1	8·4 8·7	·047	L.
			254·5	0·61	1	...	·931	B.
Σ 158 ...	1 41	57 20	259·7	1·91	1	8·3 8·8	·047	L.
			257·2	2·06	1	...	·099	B.
β 1016 ...	1 44	57 25	23·3	0·48	1	8·5 8·5	·047	L.
			26·1	0·51	1	...	·099	B.
β 235 ...	1 45	39 30	94·7	0·81	2	7·0 7·5	·504	B.
Hough 311	1 45	65 50	178·8	0·30	1	7·5 7·7	·964	B.
Σ 205 (γ^1 Andromedæ)	1 58	48 10	62·2	9·76	1	3·0 5·0	·041	L.
OΣ 38 (γ^2 Andromedæ)	1 58	48 10	114·8	0·40	2	5·0 6·2	·127	L.
Σ 208 ...	1 58	64 33	53·9	0·77	1	6·2 8·4	·958	W.B.
Hough 497	2 6	53 7	62·6	0·53	1	8·2 9·0	·041	L.
Σ 228 ...	2 8	42 59	94·2	0·55	1	6·7 7·6	·192	L.
			82·7	0·55	1	...	·964	B.
Σ 257 ...	2 18	28 55	248·5	0·37	1	7·2 7·7	·192	L.
OΣ 42 ...	2 27	38 8	121·0	0·40	1	7·0 7·5	·192	L.
OΣ 44 ...	2 36	47 44	56·8	1·33	1	7·8 8·5	·099	B.
Σ 305 ...	2 42	71 2	317·0	2·92	1	7·3 8·2	·022	B.
			313·1	3·10	3	...	·945	W.B.
β 262 ...	2 42	59 22	62·4	1·56	1	8·2 9·6	·947	W.B.
β 525 ...	2 53	52 31	137·4	0·20	1	7·5 7·5	·104	W.B.
Σ 333 ... (ε Arietis)	2 53	69 4	201·7	1·28	2	5·7 6·0	·965	W.B.
Σ 346 A.B.	3 0	65 8	93·8	0·45	2	6·0 6·0	·544	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position. Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
			87·6	0″·56	2	...	·553	B.
A.C.	...	...	351·1	5·21	1	...	·104	W.B.
β 1030 ...	3 4	68 39	155·7	0·53	1	8·4 8·4	·989	L.
β 530 ...	3 8	67 25	188·4	1·78	1	9·7 10·1	·980	L.
OΣ 53 ...	3 11	51 44	236·6	0·59	1	7·2 8·0	·142	B.
			238·2	0·58	1	...	·192	L.
β 533 ...	3 29	58 39	44·0	0·30	1	7·0 7·0	·104	W.B.
			50·4	0·57	1	...	·142	B.
Σ 412 ... (7 Tauri)	3 28	65 52	13·8	0·15	1	6·6 6·7	·104	W.B.
			13·9	0·27	1	...	·964	B.
OΣ 66 ...	3 45	49 30	139·4	0·48	1	7·5 8·0	·142	B.
			132·2	0·43	1	...	·192	L.
β 880 ...	3 38	58 9	1·8	0·62	1	8·7 8·9	·192	L.
OΣ 531 ...	4 1	52 12	128·3	2·07	1	8·5 10·2	·142	B.
OΣ 77 ...	4 9	58 34	178·0	0·22	1	7·0 7·5	·118	B.
OΣ 80 ...	4 17	47 46	189·7	0·48	1	6·5 7·0	·142	B.
			176·3	0·50	1	...	·203	W.B.
			173·9	0·60	1	...	·939	L.
Σ 535 ...	4 18	78 53	327·9	1·49	1	6·7 8·2	·022	B.
			320·2	1·36	1	...	·126	W.B.
β 550 ... (Aldebaran)	4 30	73 42	109·0	31·29	2	1 14	·131	L.
Hastings ...	4 30	70 29	55·6	0·36	1	8·0 9·0	·068	W. B.
OΣ 86 ...	4 31	70 27	60·8	0·55	1	7·5 7·5	·137	W.B.
Σ 567 ...	4 31	70 44	320·8	1·68	1	8·5 9·0	·947	W.B.
Σ 572 ...	4 32	63 16	19·6	3·86	1	6·5 6·5	·947	W.B.
β 883 ...	4 45	79 6	60·4	0·26	3	7·5 7·5	·146	L.
			62 6	0·27	1	...	·186	B.
			63·3	0·27	6	...	·211	W.B.
β 552 ...	4 46	76 31	207·9	0·45	2	6·9 10·2	·193	L.
OΣ 93 ...	4 55	85 3	51·3	0·74	2	7·5 9·0	·164	B.
OΣ 98 ...	5 2	81 39	174·4	0·98	2	5·5 7·0	·164	B.
			170·4	0·73	1	...	·214	L.
β 1047 ...	5 3	62 5	53·3	0·45	1	8·5 8·8	·214	L.
β 885 ...	5 6	91 54	25·8	0·56	1	8·3 8·4	·186	B.
β 1006 ...	5 7	92 19	204·1	0·51	1	8·5 10·9	·186	B.
β 886 ...	5 15	56 17	243·7	1·18	1	9·1 9·6	·300	L.
OΣ 105 ...	5 16	77 26	106·5	0·30	1	7·8 7·8	·186	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
Dawes 5 ...	5 19	92 29	81·4	1·24	1	3·5 5·5	·164	B.
Σ 719 ...	5 24	60 32	322·5	0·90	1	7·0 9·5	·104	W.B.
Σ 728 ...	5 26	84 7	188·8	0·38	2	5·5 6·5	·175	B.
Σ 749 ...	5 31	63 8	173·6	0·77	1	7·1 7·2	·104	W.B.
			174·8	0·91	3	...	·164	B.
β 1240 A.B.	5 32	59 34	346·4	0·28	1	5·6 6·0	·203	W.B.
Σ 753 A.C.			267·8	12·53	1	...	·203	W.B.
OΣ 112 ...	5 33	52 6	73·4	0·52	3	7·5 7·5	·190	B.
			68·1	0·55	1	...	·192	L.
β 1007 ...	5 35	73 31	254·7	0·20	2	6·0 6·2	·214	B.
Σ 774 ...	5 36	92 0	155·7	2·41	2	2·0 5·7	·175	B.
OΣ 118 ...	5 42	69 10	314·0	0·48	2	8·0 8·8	·173	W.B.
			312·2	0·54	1	...	·238	L.
Σ 799 ...	5 45	51 28	180·4	0·95	3	7·2 8·3	·190	B.
			180·8	0·85	1	...	·192	L.
Perrine ...	5 51	37 19	307·1	1·92	1	9·0 9·8	·255	L.
β 1241 ...	6 4	66 52	345·8	0·45	1	5·9 10·0	·238	L.
			329·2	0·41	1	...	·241	B.
β 1058 ...	6 4	66 59	276·8	0·36	1	7·2 7·5	·183	W.B.
			276·5	0·30	2	...	·214	B.
			268·3	0 25	1	...	·238	L.
*	6 8	67 24	87·6	2·99	1	9·5 10·0	·238	L.
β 1008 ...	6 9	67 28	296·6	0·88	1	3·0 10·8	·238	L.
Σ 881 ...	6 13	30 35	108·3	0·65	1	6·4 7·9	·307	L.
β 895 A.B.	6 14	61 31	150·7	0·24	1	8·2 8·4	·241	B.
Σ 888 A.C.	...	...	251·3	2·60	1	8·2 9·2	·241	B.
β 1020 ...	6 17	61 11	155·1	1·09	1	8·2 10·0	·104	W.B.
			164·4	1·10	1	...	·238	L.
Σ 899 ...	6 17	72 22	19·8	2·31	2	7·0 8·0	·115	W.B.
β 1192 ...	6 23	69 43	340·6	0·16	1	8·7 8·8	·241	B.
β 1021 ...	6 25	61 33	80·6	0·79	1	8·1 9·4	·238	L.
			78·4	0·55	1	...	·241	B.
Σ 932 ...	6 29	75 10	328·0	2·15	1	8·2 8·3	·047	L.
			324·6	1·94	1	...	·104	W.B.
OΣ 149 ...	6 30	62 38	274·8	0·66	2	6·5 9·0	·143	L.
			282·7	0·45	1	...	·241	B.
Σ 396 ...	6 31	31 48	263·2	1·66	1	7·0 8·7	·307	L.
Σ 945 ...	6 33	48 56	270·0	0·71	2	7·1 8·0	·177	B.
			260·9	0·78	1	...	·255	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
✶	6 34	...	80·0	2·03	1	...	·255	L.
Σ 948 A.B.	6 37	30 28	114·8	1·48	2	5·2 6·1	·623	L.
OΣ 156 ...	6 41	71 42	301·1	0·53	4	6·5 7·0	·123	B.
			305·9	0·49	2	...	·143	L.
OΣ 160 ...	6 48	68 43	171·9	1·52	1	6·8 9·8	·049	L.
β 899 ...	6 53	71 9	266·6	0·60	1	8·7 9·3	·044	B.
			274·1	0·60	1	...	·236	L.
β 900 ...	7 0	68 49	273·7	1·41	1	8·0 11·5	·236	L.
OΣ 165 ...	7 3	73 54	42·5	4·17	1	5 11	·236	L.
Σ 1037 ...	7 7	62 35	302·5	0·77	1	7·0 7·1	·301	L.
			302·1	0·75	1	...	·309	B.
β 1023 ...	7 9	63 56	294·5	0·19	1	8·4 8·5	·241	B.
			291·5	0·34	1	...	·301	L.
✶	7 9	63 56	226·5	0·72	1	9·5 10·0	·241	L.
OΣ 170 ...	7 14	80 31	110·7	1·56	3	7·0 7·5	·084	B.
Σ 1110 ...	7 28	57 53	225·2	5·71	1	2·7 3·7	·183	B.
			226·4	5·95	1	...	·192	L.
OΣ 175 ...	7 29	58 50	330·1	0·66	3	6·0 6·6	·083	B.
			329·0	0·71	1	...	·192	L.
OΣ 176 ...	7 33	89 14	200·0	1·56	1	7·3 7·3	·044	B.
Σ 1126 ...	7 34	84 28	144·2	0·81	2	7·0 7·0	·225	L.
			142·7	1·07	1	...	·233	B.
OΣ 177 ...	7 35	52 19	120·2	0·50	4	7·5 8·5	·123	B.
			119·3	0·59	1	...	·120	L.
Procyon ...	7 35	84 28	338·3	4·83	1	1 10	·236	L.
Σ 963 ...	7 44	30 26	69·5	0·51	1	5·9 7·1	·306	L.
OΣ 182 ...	7 47	86 20	30·8	1·16	2	7·0 7·5	·115	B.
Σ 1157 ...	7 49	92 31	62·8	1·10	1	8·0 8·0	·233	B.
OΣ 185 ...	7 52	88 35	13·5	0·30	2	6·8 7·1	·139	B.
β 581 ...	7 59	77 25	284·8	0·53	2	8·5 8·6	·284	L.
			286·1	0·35	3	...	·187	B.
Σ 1184 ...	8 3	51 49	341·2	26·82	1	8·0 8·5	·047	L.
			341·4	27·06	1	...	·329	B.
Σ 1187 ...	8 3	57 28	45·5	2·17	1	7·1 8·0	·047	L.
			41·8	2·00	1	...	·309	B.
Σ 1196 A.B. (ζ Cancri)	8 6	72 3	5·8	1·08	2	5·0 5·7	·237	B.
A.C.	...	...	113·6	5·10	2	5·0 6·5	·237	B.
B.C.	...	...	125·7	5·64	1	5·7 6·5	·241	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
Σ 1211 ...	8 12	50 42	130·6	0·95	1	8·7 9·2	·329	B
β 209 ...	8 37	50 50	358·3	1·39	2	8·2 8·7	·319	B.
Σ 1273 (ε Hydræ A.C.)	8 41	83 13	229·8	3·53	1	3·8 7·7	·236	L.
			230·2	3·43	1	...	·298	W.B.
Schiaparelli (ε Hydræ A.B.)	...	...	267·2	0·20	1	3·8 6·0	·236	L.
β 1068 ...	8 44	88 46	185·9	0·45	1	7·7 8·8	·241	B.
Σ 3121 ...	9 12	61 0	18·3	0·80	2	7·5 7·8	·258	L.
			18·9	0·89	1	...	·241	B.
			22·6	0·57	1	...	·263	W.B.*
OΣ 201 ...	9 18	61 39	222·7	1·41	2	7·5 9·0	·281	W.B.
			220·1	1·31	1	...	·301	L.
Σ 1348 ...	9 19	83 13	322·0	1·72	1	7·5 7·6	·312	W.B.
Σ 1356 ...	9 23	80 31	112·0	0·66	3	6·2 7·0	·261	B.
			114·3	0·62	1	...	·312	B.
OΣ 208 ...	9 45	35 28	281·4	0·18	1	5·0 5·6	·315	L.
			284·7	0·27	2	...	·367	B.
Σ 1389 ...	9 47	62 31	308·9	2·13	2	8·2 9·0	·334	W.B.
A.C. 5 ...	9 47	97 38	84·2	0·27	1	5·5 5·7	·233	B.
OΣ 215 ...	10 11	71 46	206·0	0·65	1	7·0 7·2	·236	L.
			210·1	0·91	2	...	·237	B.
Σ 1424 (γ Leonis)	10 14	69 39	118·3	3·67	1	2·0 3·5	·236	L.
			115·3	3·29	1	...	·427	B.
Σ 1426 A.B.	10 15	83 4	279·0	0·73	2	7·8 8·3	·275	B.
			278·0	0·64	1	...	·312	W.B.
A.C.	...	...	8·0	7·58	1	7·8 9·2	·312	W.B.
OΣ 216 ...	10 17	74 9	118·3	1·11	1	7·0 10·5	·236	L.
			116·5	1·33	1	...	·241	B.
Σ 1429 ...	10 20	64 53	252·2	0·75	1	8·3 8·3	·236	L.
			257·2	0·96	2	...	·305	W.B.
			249·0	0·97	1	...	·309	B.
OΣ 217 ...	10 21	72 16	157·0	0·62	1	7·3 7·8	·236	L.
			153·9	0·73	2	...	·237	B.
Σ 1457 ...	10 34	83 45	318·5	1·28	1	7·4 8·8	·309	B.
OΣ 225 A.B.	10 35	70 14	246·2	0·81	1	7·5 11·2	·345	W.B.
A.C.	...	...	345·7	6·51	1	7·5 9·8	·345	W.B.

* Quadrant certain.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Meas.	Mags.	Epoch 1900.	Obs.
OΣ 227 ...	10 36	78 44	349·4	0·48	1	7·5 8·5	·309	B.
			349·5	0·47	1	...	·312	W.B.
OΣ 228 ...	10 42	66 54	192·3	0·45	2	7·2 8·1	·271	B.
			191·2	0·49	2	...	·294	W.B.
OΣ 229 ...	10 42	48 21	317·5	0·80	2	6·7 7·1	·368	B.
β 915 ...	10 44	65 11	232·3	1·85	1	9·0 9·0	·315	L.
			234·1	1·46	1	...	·345	W.B.
Hough 48 ...	10 59	66 18	125·5	1·51	1	8·3 11	·290	W.B.
Σ 1504 ...	10 59	85 47	289·6	1·13	1	7·4 7·5	·209	B.
Hough 378...	10 59	51 2	228·6	0·33	1	8 8·1	·309	B.
Σ 1517 ...	11 9	69 19	272·4	0·35	2	7·3 7·3	·271	B.
			268·1	0·42	2	...	·293	W.B.
Σ 1523 (ξ Ursæ Maj.)	11 13	57 54	155·5	2·30	1	4·0 4·9	·301	L.
			151·7	2·17	2	...	·368	B.
Σ 1524 (ν Ursæ Maj.)	11 13	56 22	147·4	7·73	1	3·7 10·1	·301	L.
			147·4	7·37	1	...	·309	B.
Σ 1534 ...	11 16	71 14	324·7	5·09	1	8·0 11·0	·290	W.B.
Lalande 21846	11 24	58 57	6·7	0·94	1	7·0 11·0	·301	L.
			6·0	0·84	1	...	·312	W.B.
OΣ 234 ...	11 26	58 1	309·4	0·26	1	7·0 7·4	·427	B.
OΣ 235 ...	11 27	28 22	120·9	0·58	1	6·0 7·3	·427	B.
Σ 1554 ...	11 31	76 36	259·9	0·86	2	8·9 9·1	·329	W.B.
Σ 1555 ...	11 31	61 40	347·3	0 37	2	6·4 6·8	·294	W.B.
			347·0	0·38	2	...	·352	L.
			354·7	0·27	1	...	·309	B.
β 603 ...	11 45	75 5	315·6	0·84	1	6·4 10·3	·435	L.
Σ 1606 ...	12 5	49 31	325·8	0·97	1	6·2 7·0	·427	B.
Σ 1639 ...	12 19	63 52	173·7	0·23	3	6·7 7·9	·300	W.B.
			180·1	0·15	3	...	·340	L.
Σ 1643 ...	12 22	62 25	38·1	2·02	3	8·4 8·7	·300	W.B.
Σ 1647 ...	12 25	79 44	224·2	1·36	2	7·5 7·8	·340	W.B.
Σ 1658 ...	12 30	82 0	356·5	2·60	1	8·0 9·8	·345	W.B.
Σ 1661 ...	12 30	78 3	238·3	2·50	2	8·5 8·5	·340	W.B.
Σ 1663 ...	12 32	68 15	106·0	0·46	1	7·8 8·7	·315	L.
			102·0	0·63	3	...	·323	W.B.
Σ 1670 (γ Virginis)	12 36	90 54	149·1	5·78	2	3·0 3·0	·453	B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Meas.	Mags.	Epoch 1900.	Obs.
Σ 1687 (35 Comæ Ber.)	12 48	68 13	82·3	0·99	2	5·0 7·8	·375	L.
OΣ 256 ...	12 51	90 25	77·4	0·57	1	7·2 7·6	·460	B.
β 112 ...	12 57	71 5	294·0	2·03	1	6·5 8·5	·435	L.
Σ 1711 ...	12 58	75 59	342·6	0·87	1	8·5 9·0	·345	W.B.
			347·1	0·95	1	...	·435	L.
✶	13 2	62 31	198·9	0·48	1	9·0 9·5	·345	W.B.
OΣ 260 ...	13 3	62 31	114·7	0·55	1	7·9 8·3	·345	W.B.
			124·5	0·55	2	...	·359	L.
β 929 ...	12 59	93 7	223·6	0·54	2	6·2 6·2	·444	B.
Σ 1728 (42 Comæ Ber.)	13 5	71 57	208·8	0·19	1	6·0 6·0	·345	W.B.
			196·4	0·24	2	...	·419	L.
			205·0	0·33	2	...	·444	B.
OΣ 261 ...	13 7	57 22	344·9	1·51	1	7·0 7·5	·435	L.
			345·8	1·54	1	...	·528	B.
Σ 1734 ...	13 15	86 30	195·0	1·04	1	7·2 7·9	·460	B.
Ho. 260 ...	13 19	60 15	323·2	0·52	1	8·3 8·5	·334	W.B.
			318·6	0·77	2	...	·419	L.
OΣ 266 ...	13 23	73 41	334·1	1·48	1	7·3 7·8	·334	W.B.
OΣ 269 ...	13 28	54 34	218·5	0·30	1	6·5 7·0	·435	L.
			204·2	0·34	2	...	·478	B.
Σ 1768 (25 Canum Ven.)	13 23	53 10	134·2	0·94	1	5·7 7·6	·435	L.
			129·7	0·88	2	...	·646	B.
β 612 ...	13 34	78 45	227·3	0·31	1	6·3 6·5	·427	B.
Σ 1785 ...	13 45	62 30	276·0	1·38	2	7·2 7·5	·438	W.B.
Σ 1808 ...	14 5	62 54	74·5	2 90	2	8·0 9·0	·382	W.B.
OΣ 278 ...	14 8	45 21	80·3	0·29	2	7·5 7·5	·494	B.
			93·8	0·32	2	...	·537	W.B.
β 1272 ...	14 14	40 47	135·2	1·62	1	8·4 9·5	·621	L.
β 1273 ...	14 15	41 35	191·8	...	1	8·6 10·8	·542	W.B.
			187·6	1·07	1	...	·621	L.
Σ 1863 ...	14 34	...	88·3	0·65	3	7·1 7·4	·548	B.
Σ 1865 ... (ζ Boötis)	14 36	75 49	148·6	0·21	3	3·5 3·9	·426	L.
			143·4	0·31	5	...	·482	B.
Σ 1867 ...	14 36	58 15	17·7	1·06	1	7·7 8·2	·460	B.
			12·4	1·09	2	...	·487	W.B.
Σ 1871 ...	14 38	38 11	293·2	1·92	3	7·0 7·0	·548	B.
Σ 1877 ... (ε Boötis)	14 38	62 29	326·6	2·81	1	3·0 6·3	·465	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
OΣ 285 ...	14 42	47 11	124·5	0·26	2	7·1 7·6	·453	B.
			130·3	0·34	...	...	·520	W.B.
Σ 1883 ...	14 44	83 38	242·2	0·57	1	7·0 7·0	·460	B.
Σ 1888 ... (ξ Boötis)	14 47	70 27	198·2	2·98	1	5·1 7·4	·441	L.
OΣ 287 ...	14 47	44 38	318·6	0·77	2	7·5 7·6	·453	B.
			302·8	0·63	2	...	·520	W.B.
β 31 ...	14 48	70 50	189·9	1·44	1	8·4 9·7	·441	L.
OΣ 288 ...	14 49	73 53	189·2	1·70	2	6·5 7·0	·422	L.
			194·2	1·67	1	...	·465	W.B.
Σ 1908 ...	15 1	55 7	145·2	1·30	1	8·2 9·2	·435	L.
			143·4	1·54	2	...	·562	W.B.
Hough 60 ...	15 10	54 44	32·7	0·43	1	8 8	·435	L.
			37·2	0·23	1	...	·446	B.
			46·5	0·24	3	...	·538	W.B.
Σ 1932 ...	15 14	62 48	328·2	0·71	1	5·5 6·0	·430	W.B.
			329·2	0·80	2	...	·437	B.
			332·3	0·66	2	...	·471	L.
Hough 62 ...	15 17	54 40	107·4	1·09	1	8·2 8·3	·435	L.
			292·3	1·20	2	...	·520	W.B.
Σ 1937 ... (η Cor. Bor.)	15 19	59 21	6·0	0·68	2	5·0 5 7	·419	L.
			1·3	0·63	3	...	·426	B.
Σ 1938 ...	15 20	52 16	71·6	0·94	1	4·0 6·7	·575	W.B.
			69·3	0·86	2	...	·589	B.
OΣ 296 ...	15 22	45 38	305·1	1·50	2	7·0 8 6	·676	L.
*	15 25	43 27	340·4	2·88	1	7·5 9·0	·715	L.
Σ 1957 ...	15 31	76 42	153·1	1·07	1	8·0 9·5	·430	W.B.
OΣ 298 ...	15 32	49 50	180·6	1·03	3	7·0 7·3	·658	L.
			179·4	0·94	2	...	·689	B.
Σ 1967 ... (γ Cor. Bor.)	15 39	63 23	116·1	0·48	3	4·0 7·0	·426	B.
			117·1	0·44	1	...	·501	L.
β 619 ...	15 39	76 1	7·9	0·48	3	6·5 7·0	·393	B.
β 621 ...	15 41	44 58	55·5	0·63	1	7·5 8·0	·629	L.
			64·4	0·42	1	...	·657	B.
β 810 ...	15 48	47 14	89·0	1·08	1	8·6 11·2	·637	L.
OΣ 303 ...	15 56	76 27	148·8	0·83	2	7·4 7·9	·427	B.
			142·4	0·65	1	...	·430	W.B.
β 355 ...	16 5	44 21	280·7	0·41	2	7·9 9·2	·668	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
Σ 2032 ...	16 2	55 54	210·3	4·36	1	5·0 6·1	·715	L.
β 951 ...	16 20	56 23	51·9	0·85	2	8·2 8·7	·668	B.
Σ 2054 ...	16 23	28 4	355·3	1·07	2	5·7 6·9	·668	B.
Σ 2052 ...	16 25	71 28	91·9	1·93	1	7·5 7·5	·430	W.B.
			93·3	1·86	1	...	·446	B.
			93·2	1·86	1	...	·501	L.
β 813 ...	16 24	63 15	168·7	1·04	2	8·5 8·5	·367	B.
			167·6	0·98	1	...	·430	W.B.
β 814 ...	16 24	49 53	316·5	0·32	2	8·4 8·7	·668	B.
β 817 ...	16 28	66 33	149·7	0·96	2	8·2 8·2	·367	B.
			145·9	1·09	1	...	·501	L.
β 818 ...	16 30	59 17	34·7	3·87	1	6·3 13·5	·643	L.
Σ 2084 ...	16 38	58 13	239·5	0·80	6	3·0 6·5	·603	L.
(ζ Herculis)			232·1	0·63	2	...	·685	B.
			224·1	0·95	1	...	·993	L.
✶	16 38	45 20	121·6	5·68	1	9 11	·640	L.
Σ 2091 ...	16 39	48 37	305·7	1·07	3	7·5 8·0	·580	B.
De. 15 ...	16 40	46 20	315·2	0·51	3	8·0 8·5	·580	B.
Σ 2106 ...	16 46	80 25	310·0	0·27	1	6·7 8·4	·548	W.B.
Σ 2107 ...	16 48	61 10	312·4	0·35	1	6·5 8·0	·543	W.B.
			324·3	0·20	2	...	·630	L.
			319·7	0·31	2	...	·668	B.
β 821 ...	16 48	57 59	318·2	1·15	1	8·4 8·9	·405	B.
			311·4	1·29	2	...	·492	L.
Σ 2118 ...	16 57	24 50	93·2	0·16	1	6·4 6·9	·731	B.
Σ 3127 ...	17 11	65 3	192·6	14·63	1	3·0 8·1	·660	L.
OΣ 327 ...	17 12	33 45	313·0	0·17	1	7·6 8·0	·731	B.
β 629 ...	17 15	57 50	344·0	0·99	2	8·4 8·8	·572	L.
β 45 ...	17 15	57 26	290·4	4·97	2	8·6 8·8	·512	L.
β 628 ...	17 16	57 14	355·8	0·50	3	8·8 9·4	·596	L.
✶	17 16	57 27	28·3	5·55	1	9·5 10·0	·523	L.
✶	17 16	57 25	302·1	3·23	2	9·0 9·5	·512	L.
β 630 ...	17 17	57 36	224·1	1·39	2	8·6 10·2	·572	L.
β 1250 ...	17 21	59 9	66·2	2·27	1	9·4 9·5	·523	L.
Σ 2173 ...	17 25	90 56	329·5	1·01	3	5·8 6·3	·653	B.
β 1121 ...	17 33	77 24	239·0	0·66	1	8·5 9·0	·523	L.
β 631 ...	17 35	90 36	67·1	0·36	2	7·5 7·6	·641	B.
Σ 2199 ...	17 37	34 12	88·8	1·71	1	7·2 7·8	·630	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
β 1251 ...	17 37	74 D	76·7	1·13	2	6·0 11·5	·570	L.
Σ 2203 ...	17 38	48 18	324·0	0·65	1	7·5 7·8	·630	L.
			317·4	0·68	2	...	·798	B.
Σ 2205 ...	17 41	72 15	305·7	1·97	3	8·3 8·6	·600	L.
Σ 2215 ...	17 42	72 16	297·2	0·63	2	5·9 7·9	·581	L.
			295·2	0·67	1	...	·731	B.
Hough 70 ...	17 42	59 25	108·4	0·30	2	8 8	·798	B.
OΣ 337 ...	17 46	82 44	270·3	0·39	3	7·5 8·0	·653	B.
OΣ 338 ...	17 47	74 39	18·6	0·69	3	6 5 7·0	·652	B.
			15·5	0·75	1	...	·698	L.
*	17 47	74 39	354·5	1·26	1	10·0 10·5	·698	L.
A.C. 8 ...	17 19	60 18	58·4	0·32	1	8·0 9·8	·865	B.
A.C. 9 ...	17 50	60 10	55·4	0·95	1	8·4 8·7	·865	B.
β 1125 ...	17 57	88 40	19·0	1·00	1	5·1 9·1	·671	L.
Σ 2262 ... (τ Ophiuchi)	17 58	98 11	257·7	1·29	1	5·0 5·7	·671	L.
Σ 2272 (70 Ophiuchi)	18 0	87 28	246·9	1·61	9	4·1 6·1	·566	W.B.
OΣ 341 ...	18 1	68 34	77·0	0·26	2	6·4 7·7	·681	B.
OΣ 343 ...	18 2	41 52	87·5	2·61	1	7·2 10·2	·638	L.
Σ 2281 ...	18 4	86 2	224·8	0·31	3	5·7 7·2	·572	W.B.
Σ 2283 ...	18 4	83 52	80·3	1·11	2	7·2 7·7	·537	W.B.
Σ 2289 ...	18 5	73 32	233·6	1·20	3	6·5 7·0	·548	W.B.
Σ 2294 ...	18 9	89 51	100·3	0·19	1	7·4 7·7	·624	B.
β 1091 ...	18 9	51 26	35·3	0·29	2	8·6 8·6	·798	B.
β 641 ...	18 17	68 33	344·1	0·96	2	7·3 9·0	·584	W.B.
			347·0	0·95	2	...	·687	L.
*	18 18	69 26	110·8	6·51	1	10 11	·698	L.
Σ 2315 ...	18 21	62 39	203·9	0·26	2	7·0 8·0	·589	L.
			212·9	0·17	1	...	·543	W.B.
Σ 2318 ...	18 21	64 4	252·8	21·12	1	7·9 10·0	·465	W.B.
β 1203 ...	18 21	89 15	68·0	0·22	1	7·5 7·7	·624	B.
			90·2	0·20	1	...	·676	L.
OΣ 543 ...	18 22	43 11	132·9	1·01	2	8·0 10·0	·688	L.
			128·6	1·11	2	...	·798	B.
OΣ 351 ...	18 23	41 19	18·0	0·52	2	7·0 7·0	·688	L.
			20·8	0·56	2	...	·798	B.
OΣ 354 ...	18 27	83 18	165·2	0·70	2	7 8	·574	W.B.
Hough 86 ...	18 30	54 54	181·7	0·29	1	8 9	·660	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Meas.	Mags.	Epoch 1900.	Obs.
O Σ 357 ...	18 31	78 24	242·4	0·33	2	8·2 8·2	·574	W.B.
			234·8	0·47	1	...	·676	L.
O Σ 358 ...	18 31	73 12	192·4	1·95	3	6·8 7·2	·513	W.B.
			190·2	1·77	...	...	·676	L.
Hough 87...	18 34	73 34	103·1	0·38	2	7·8 8·1	·574	W.B.
			102·1	0·39	1	...	·676	L.
*	18 34	61 18	257·7	1·11	2	9·5 10·0	·597	W.B.
			262·0	0·92	1	...	·698	L.
*	18 22	63 57	309·6	5·56	1	10 10	·698	L.
Σ 2356 ...	18 34	61 23	60·4	0·90	3	8·2 8·6	·586	W.B.
			60·8	0·98	1	...	·698	L.
*	18 35	61 22	30·1	6·27	1	9 9	·698	L.
Σ 2362 ...	18 35	54 2	2·6	4·08	1	7·1 8·4	·660	L.
Hough 437 A.D.	18 37	58 27	301·4	0·32	1	8 8	·643	L.
C.D.	...	...	339·0	4·14	1	...	·643	L.
Σ 2367 ...	18 37	59 48	85·7	0·18	2	7·2 7·6	·672	L.
			83·5	0·17	3	...	·740	B.
Σ 2369 ...	18 39	87 30	92·5	0·95	1	8 8	·624	B.
Σ 2400 A.B.	18 44	73 51	188·3	1·88	1	8·0 11·1	·709	L.
A.C.	...	...	178·3	2·92	1	8·0 11·0	·619	W.B.
	...	...	188·6	2·79	1	...	·709	L.
B.C.	...	...	188·2	0·92	1	11·1 11·0	·709	L.
A.D.	...	...	157·1	0·39	1	8·0 9·5	·709	L.
Σ 2402 ...	18 45	79 25	205·8	0·82	3	8·0 8·4	·626	W.B.
β 971 ...	18 45	40 41	8·3	0 27	1	6·5 8·5	·731	B.
β 1255 ...	18 52	41 15	89·7	1·43	1	5·8 12·5	·638	L.
β 648 ...	18 53	57 16	224·2	1·17	2	6·0 9·2	·622	L.
			228 9	1·10	1	...	·731	B.
Σ 2422 ...	18 53	63 54	91·2	0·91	3	7·9 8·2	·587	W.B.
β 649 ...	18 55	57 40	12·2	1·43	1	8·2 10·6	·731	B.
Σ 2437 ...	18 57	70 59	62·7	0·71	3	7·8 8·2	·640	W.B.
Σ 2454 ...	19 2	59 43	252·4	0·96	2	8 9	·674	W.B.
			254·2	0·80	1	...	·731	B.
*	19 2	60 5	190·1	1·03	1	9 10	·501	L.
Σ 2455 ...	19 3	67 59	80·2	3·39	4	7 8	·635	W.B.
Σ 2488 ...	19 11	70 9	326·2	1·58	1	8·5 9·7	·772	W.B.
O Σ 368 ...	19 11	74 1	217·2	0·74	3	8 9	·644	W.B.
			213·8	0·95	1	...	·671	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Meas.	Mags.		Epoch 1900.	Obs.
OΣ 371 ...	19 12	62 42	153·0	0·88	3	7	7	·644	W.B.
β 1256 ...	19 14	83 51	36·5	0·63	1	8·3	8·3	·624	B.
			34·0	0·63	1	...		·671	L.
H.A.H. 95...	19 15	87 14	160·4	0·40	1	6·0	6·5	·624	B.
			149·8	0·28	1	...		·671	L.
β 141 ...	19 18	67 42	77·0	0·72	1	7·5	8·5	·676	B.
β 1129 ...	19 19	37 50	329·1	0·28	1	6·3	6·3	·865	B.
Σ 2525 ...	19 22	62 52	321·7	0·47	2	8·0	8·0	·619	B.
OΣ 375 ...	19 30	72 7	150·4	0·57	2	8·0	9·0	·619	B.
			149·0	0·28	1	...		·772	W.B.
Σ 2544 ...	19 32	81 56	202·9	1·00	1	7·8	9·5	·671	L.
Σ 2556 ...	19 35	68 0	145·7	0·45	2	7·3	7·8	·619	B.
β 1132 ...	19 39	63 18	211·3	0·53	1	8·3	8·7	·624	B.
*	19 40	63 6	349·4	0·33	1	...		·624	B.
β 658 ...	19 40	62 9	297·6	0·50	2	6·7	9·7	·619	B.
Σ 2579 ... (δ Cygni)	19 42	45 7	300·4	1·44	2	3·0	7·9	·619	B.
A.G.C. 11 ...	19 44	71 7	162·9	0·20	2	4·5	6·0	·619	B.
OΣ 387 ...	19 45	54 58	334·1	0·57	1	7·2	8·2	·778	B.
Hough 580	19 48	67 48	271·7	0·78	2	8·0	8·1	·696	B.
			279·7	0·54	1	...		·772	W.B.
Σ 2600 ...	19 51	67 46	58·3	3·20	2	8·3	9·7	·710	W.B.
			54·8	3·06	1	...		·756	B.
β 425 A.B.	19 53	69 59	238·6	1·43	1	8·4	8·5	·756	B.
			242·6	1·41	1	...		·772	W.B.
A.C.			41·8	20·22	1		11·9	·772	W.B.
A.C. 16 ...	19 54	63 2	56·1	0·44	2	7	9	·696	B.
			56·6	0·37	1	...		·660	L.
			54·0	0·49	1	...		·772	W.B.
Σ 2607 A.C.	19 55	48 0	291·8	3·24	1	7·2	9·2	·778	B.
OΣ 392 A.B.			300·7	0·28	1	7·2	9·0	·778	B.
OΣ 395 ...	19 58	65 21	275·8	0·66	1	5·8	6·2	·709	L.
			103·8	0·55	2	...		·739	B.
*	19 59	65 22	20·1	0·45	1	8	9	·709	L.
Σ 2624 A.B.	19 0	54 16	173·6	1·56	1	7·2	7·8	·719	W.B.
β 57 ...	20 1	74 47	119·7	2·38	1	6·7	11·3	·671	L.
Hough 128	20 20	47 20	26·0	1·02	1	6·3	10·9	·660	L.
*	20 20	47 11	176·0	1·55	1	8	9	·660	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Meas.	Mags.	Epoch 1900.	Obs.
Σ 2695 ...	20 28	64 39	80·6	1·04	3	6·2 8·0	·630	W.B.
β 670 ...	20 28	76 24	45·1	0·39	1	8·5 8·9	·671	L.
*	20 28	76 24	143·0	0·31	1	9·0 9·5	·671	L.
β 151 (β Delphini)	20 33	75 45	9·0	0·64	1	4·5 6·0	·613	B.
			9·8	0·63	1	...	·671	L.
Hough 152...	21 8	62 7	329·6	0·53	1	8·5 9·5	·616	L.
			319·5	0·52	3	...	·819	W.B.
OΣ 535 ...	21 10	80 27	206·0	0·33	1	4·1 4·1	·613	B.
Σ 2799 ...	21 24	79 21	295·3	1·41	3	6·6 6·6	·612	W.B.
Σ 2804 ...	21 28	69 44	332·1	3·11	3	7·3 8·0	·612	W.B.
β 1212 ...	21 34	90 33	261·8	0·51	1	6·5 6·9	·613	B.
Hough 165...	21 38	71 29	76·5	0·35	2	8·0 8·2	·794	W.B.
Σ 2822 ...	21 39	61 43	126·6	2·66	1	4·0 5·0	·019	L.
			123·4	2·59	3	...	·790	W.B.
Hough 166...	21 40	62 38	86·9	0·24	1	7·5 7·5	·613	B.
			92·4	0·29	2	...	·655	L.
			93·2	0·31	1	...	·739	W.B.
Σ 2824 (κ Pegasi A.C.)	21 40	64 49	295·7	12·77	1	3·9 10·8	·537	W.B.
β 989 (κ Pegasi A.B.)	21 40	64 49	259·2	0·14	5	3·9 5·0	·555	L.
			251·2	0·14	4	...	·689	W.B.
			253·3	0·16	1	...	·613	B.
*	21 41	64 55	295·3	4·36	1	9·5 10·0	·694	L.
Hough 171...	21 48	62 41	170·6	0·68	1	8 8	·613	B.
			169·4	0·76	2	...	·655	L.
			159·7	0·81	1	...	·904	W.B.
β 75 ...	21 50	79 35	43·3	0·95	1	8·1 8·3	·756	B.
			46·7	0·89	1	...	·816	W.B.
Σ 2849 ...	21 53	76 19	266·8	1·43	1	8·2 10·7	·694	L.
			265·6	1·53	2	...	·767	W.B.
Hough 179	22 8	60 26	257·3	0·57	2	8·0 8·5	·655	L.
			259·4	0·52	1	...	·756	B.
			260·1	0·41	1	...	·958	W.B.
Σ 2881 ...	22 10	60 57	93·8	1·47	1	7·7 8·2	·616	L.
			98·5	1·33	3	...	·714	W.B.
Hough 180	22 12	46 38	221·2	0·55	1	7·0 7·5	·931	B.
β 1216 ...	22 15	61 0	318·1	0·53	1	8·4 8·7	·694	L.
			310·0	0·49	1	...	·756	B.
			315·4	0·68	2	...	·944	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Meas.	Mags.	Epoch 1900.	Obs.
	h m	° ′	°	″				
Σ 2900 ...	22 19	69 40	176·6	1·54	1	6·0 9·2	·717	W.B.
β 1218 ...	22 23	60 50	52·5	1·42	2	8·6 8·8	·811	W.B.
			53·6	1·45	1	...	·756	B.
Hough 296	22 36	75 59	57·3	0·32	2	5·5 5·5	·898	B.
Σ 2934 ...	22 37	69 6	143·8	0·83	1	8·2 9·2	·694	L.
			138·3	0·85	3	...	·838	W.B.
β 1144 ...	22 38	60 18	83·5	0·28	1	9·5 9·5	·790	L.
β 710 ...	22 38	60 50	235·5	0·36	1	8·0 8·5	·790	L.
β 711 ...	22 40	79 20	47·9	1·07	1	9·0 10·2	·730	L.
ΟΣ 483 ...	22 54	78 48	221·9	0·81	2	6·0 7·5	·743	W.B.
			228·6	0·94	1	...	·756	B.
β 1025 ...	23 9	77 53	277·2	0·58	1	8·0 10·8	·739	W.B.
β 79 ...	23 12	92 5	85·4	0·70	1	8·0 8·5	·931	B.
β 80 ...	23 14	85 10	10·7	0·27	1	8·0 8·5	·931	B.
β 720 ...	23 29	59 14	168·7	0·39	1	5·5 5·5	·790	L.
			159·3	0·31	1	...	·964	B.
			168·3	0·34	1	...	·983	W.B.
β 858 ...	23 36	58 0	266·0	0·56	2	8·0 8·2	·948	B.
			266·4	0·58	1	...	·983	W.B.
Σ 3050 ...	23 54	56 51	213·8	2·40	1	6·0 6·0	·816	W.B.
Σ 3056 ...	23 59	56 20	147·8	0·52	2	7·4 7·4	·477	B.
			143·0	0·52	1	...	·958	W.B.

Results of Double-star Measures with the 8-inch Equatorial at Windsor, New South Wales, in the Years 1899 and 1900. By John Tebbutt.

Ref. No.	Star.	Observed Mags.	Approx. Place of Star at Beginning of Year. R.A. h m	Dec. S. ° ′	Time of Obs. 1899+	Position Angle. °	Distance. ″	No. of Obs.	Mag. power.	Eyes.	Hour-angles h m	h m	Weight 1 to 5.
1	λ Toucani	7, 8	0 48·6	70 3	1·071	81·1	21·00	10–6	138	R	4 45 W	5 13 W	3
2	ζ Phœnicis	4½, 8	1 4·2	55 47	1·156	243·5	6·72	10–6	138	R	4 23 W	4 46 W	3
3	P.I. 127	6, 8	1 31·5	30 25	1·060	94·4	2·02	10–7	230	R	2 43 W	3 11 W	3
4	,,	6. 8	,,	,,	1·068	92·4	2·22	10–7	230	R	3 2 W	3 31 W	4
5	,,	...	,,	,,	1·071	93·6	2·09	10–6	230	R	3 18 W	3 42 W	3
6	*p* Eridani	...	1 36·0	56 42	1·047	223·0	8·33	10–8	230	R	2 18 W	2 58 W	3
7	,,	...	,,	,,	1·049	222·3	7·81	10–6	230	R	1 41 W	2 22 W	3
8	,,	...	,,	,,	1·060	223·7	8·22	10–7	230	R	1 40 W	2 20 W	2
9	θ Eridani	5, 6	2 54·5	40 42	1·140	87·3	8·09	6–5	230	R	3 39 W	4 2 W	2
10	,,	4½, 5½	,,	,,	1·145	87·7	8·47	10–6	230	R	...	2 55 W	3
11	*h* 3556	7. 9	3 8·9	44 48	1·077	219·7	2·52	10–5	138	P	2 37 W	3 21 W	3
12	,,	...	,,	,,	1·085	215·5	1·89	10–3	230	R	2 53 W	3 34 W	3
13	,,	6½, 9	,,	,,	1·085	219·1	2·63	10–3	138	P	2 53 W	3 34 W	4
14	*h* 3586_3	6½, 7	3 36·1	60 6	1·156	270·8	57·43	10–4	138	R	2 39 W	3 7 W	3
15	*h* 3586_2	7½, 8	3 36·2	40 41	1 145	326·9	7·75	10–6	230	P	3 13 W	...	4

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Ref No.	Star.	Observed Mags.	Approx. Place of Star at Beginning of Year. R.A. h m	Dec. S. ° ′	Time of Obs. 1899+	Position Angle. °	Distance. ″	No. of Obs.	Mag. power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
16	h 3586$_2$	7½, 8	3 36·2	40 41	1·151	327·3	7·68	10–6	138	P	3 20 W	3 57 W	4
17	f Eridani	6, 6½	3 44·9	37 56	1·151	207·6	7·46	10–6	138	P	4 0 W	4 19 W	3
18	,,	7½, 8	,,	,,	1·153	208·2	7·23	7–5	138	R	1 44 W	2 14 W	4
19	θ Reticuli	7, 8	4 16·6	63 30	1·156	2·4	3·93	10–6	138	P	2 37 W	3 4 W	4
20	ι Pictoris	6, 6½	4 48·7	53 38	1·175	57·8	12·15	10–5	138	R	3 14 W	3 37 W	3
21	,,	6, 6½	,,	,,	1·184	58·0	11·70	10–5	138	R	1 23 W	1 50 W	3
22	h 3823	8, 8	5 56·6	31 3	1·233	112·0	2·83	10–6	300	R	3 43 W	4 12 W	4
23	,,	...	,,	,,	1·236	112·7	2·60	10–6	300	R	3 22 W	3 52 W	3
24	,,	8, 8	,,	,,	1·299	115·2	2·69	10–8	138	R	2 59 W	3 32 W	3
25	Lacaille 2145	...	6 2·2	48 27	1·156	42·2	...	10	300	R	1 30 W	1 45 W	2
26	,,	7, 7	,,	,,	1·175	41·8	...	10	300	R	1 6 W	1 35 W	3
27	,,	7, 7	,,	,,	1·225	38·8	1·77	10–8	300	P	3 19 W	3 47 W	4
28	,,	...	,,	,,	1·227	38·8	1·42	7–5	300	P	3 51 W	4 23 W	2
29	,,	...	,,	,,	1·233	39·8	1·93	10–7	300	R	2 22 W	2 57 W	4
30	,,	...	,,	,,	1·233	39·2	1·96	10–6	300	P	2 57 W	3 22 W	5
31	γ Argûs A.B.	2, 3½	8 6·5	47 2	1·101	219·9	40·09	10–5	138	R	1 58 E	1 9 E	3
32	,,	2, 3½	,,	,,	1·112	219·6	41·23	10–5	138	P	3 24 E	2 55 E	3
33	γ Argûs A.C.	2, 7	,,	,,	1·112	151·4	62·05	7–4	...	P	2 21 E	1 46 E	3

Ref. No.	Star.	Observed Mags.	Approx. Place of Star at Beginning of Year. R.A. h m	Dec. S. ° ′	Time of Obs. 1899+	Position Angle. °	Distance. ″	No. of Obs.	Mag. power.	Eyes.	Hour-angles. h m	h m	Weight 1 to 5.
34	γ Argûs A.C.	2½, 8	8 6·5	47 2	1·126	151·8	62·08	10–5	138	P	2 23 E	1 49 E	3
35	α Crucis	...	12 21·0	62 33	1·400	117·6	4·90	10–10	300	P	4 50 E	4 28 E	4
36	,,	...	,,	,,	1·414	119·9	5·16	10–8	300	R	2 33 E	2 9 E	4
37	γ Centauri	...	12 36·0	48 24	0·332	356·5	1·60	10–10	300	P	2 14 E	1 43 E	4
38	,,	...	,,	,,	0·334	357·2	1·60	10–8	300	P	1 58 E	1 32 E	3
39	,,	4, 4	12 36·0	48 25	1·299	357·6	1·84	10–10	300	P	2 28 E	2 2 E	4
40	,,	...	,,	,,	1·301	358·6	1·84	10–10	300	P	3 35 E	3 8 E	4
41	,,	4, 4	,,	,,	1·301	358·5	1·77	10–8	300	P	3 8 E	2 49 E	5
42	,,	4, 4	,,	,,	1·304	356·2	1·57	10–7	535	P	1 56 E	1 30 E	5
43	,,	4, 4	,,	,,	1·310	358·2	1·51	10–8	535	P	3 41 E	3 17 E	5
44	,,	4, 4	,,	,,	1·329	356·8	1·48	10–7	535	P	2 38 E	2 14 E	4
45	γ Virginis	...	12 36·6	0 54	1·400	150·3	5·95	10–8	300	P	3 19 E	2 56 E	3
46	,,	...	,,	,,	1·414	151·1	6·09	10–8	300	R	1 12 E	0 41 E	5
47	,,	...	,,	,,	1·416	151·4	5·97	10–8	535	R	1 10 E	0 45 E	5
48	,,	...	,,	,,	1·422	148·8	6·00	10–10	535	P	2 58 E	2 29 E	4
49	β Muscæ	4, 4	12 40·1	67 34	1·301	341·9	1·42	10–10	300	R	2 33 E	1 51 E	5
50	,,	4, 4	,,	,,	1·304	342·8	1·08	10–10	535	P	2 54 E	2 15 E	5
51	,,	4, 4	,,	,,	1·329	343·1	...	10	535	R	2 7 E	1 39 E	3

Ref. No.	Star.	Observed Mags.	Approx. Place of Star at Beginning of Year. R.A.	Dec. S.	Time of Obs.	Position Angle.	Distance.	No. of Obs.	Mag. power.	Eyes.	Hour-angles.		Weight 1 to 5.
			h m	°	1899+	°	″				h m	h m	
52	β Muscæ	...	12 40·1	67 34	1·332	341·0	1·30	10–8	300	P	2 51 E	2 9 E	3
53	,,	...	,,	,,	1·332	343·0	...	10	535	P	2 51 E	2 9 E	4
54	,,	$4\frac{1}{2}$, $4\frac{1}{2}$	,,	,,	1·419	339·2	...	10	300	R	0 30 E	0 5 E	3
55	*h* 4634	8, 9	13 51·5	55 29	1·332	26·3	14·35	10–6	138	P	2 59 E	2 28 E	5
56	,,	...	,,	,,	1·334	26·2	14·31	10–10	138	P	4 25 E	3 57 E	5
57	Arg. G. Cat. 19385	8, 8	14 14·5	41 59	1·425	215·3	1·98	10–5	138	R	1 51 E	1 16 E	3
58	α Centauri	...	14 32·7	60 25	0·323	209·0	21·74	10–10	300	P	4 23 E	3 36 E	3
59	,,	...	,,	,,	0·332	210·6	21·99	10–10	300	R	5 6 E	4 34 E	2
60	,,	...	,,	,,	0·334	209·7	21·90	10–6	300	P	3 21 E	2 54 E	3
61	,,	...	,,	,,	0·337	209·5	22·14	10–8	300	P	3 29 E	3 0 E	3
62	,,	...	,,	,,	0·340	209·8	22·21	10–10	300	P	2 56 E	2 27 E	3
63	,,	...	14 32·8	60 25	1·301	210·7	21·87	10–10	300	P	3 32 E	2 57 E	3
64	,,	...	,,	,,	1·304	210·8	...	10	535	R	5 38 E	4 57 E	4
65	,,	...	,,	,,	1·304	210·7	22·06	10–8	535	P			4
66	,,	...	,,	,,	1·334	209·9	21·79	10–10	535	P	4 8 E	3 39 E	5
67	,,	...	,,	,,	1·356	210·8	21·90	10–7	535	P	2 45 E	2 14 E	3
68	,,	...	,,	,,	1·384	210·2	22·44	10–6	300	P	3 51 E	3 25 E	3
69	π Lupi	5, 5	14 58·3	46 40	1·416	88·1	1 17	10–8	300	R	2 54 E	2 26 E	5

Ref. No.	Star.	Observed Mags.	Approx. Place of Star at Beginning of Year. R.A.	Dec. S.	Time of Obs.	Position Angle.	Distance.	No. of Obs.	Mag. power.	Eyes.	Hour-angles.		Weight 1 to 5.
			h m	° ′	1899+	°	″				h m	h m	
70	π Lupi	...	14 58·3	46 40	1·419	86·2	...	10	300	R	2 10 E	1 47 E	3
71	,,	...	,,	,,	1·422	86·2	1·57	10–5	300	R	0 30 E	0 8 E	5
72	,,	...	,,	,,	1·425	85·9	1·78	10–7	300	R	1 50 E	1 26 E	4
73	Antares	1, 7½	16 23·3	26 13	1·400	276·9	3·44	10–8	300	R	2 36 E	1 57 E	4
74	,,	1, 7½	,,	,,	1·422	275·1	3·33	10–10	300	R	2 29 E	2 4 E	4
75	,,	1, 8	,,	,,	1·425	275·7	3·23	10–8	300	R	2 41 E	2 18 E	4
76	β 416	6½, 8	17 12·1	34 53	1·416	300·1	1·42	10–6	300	P	4 25 E	3 58 E	4
77	,,	6½, 8	,,	,,	1·422	298·7	2·07	10–7	300	P	2 14 E	1 49 E	4
78	,,	6½, 7½	,,	,,	1·425	298·2	2·36	10–8	300	R	3 0 E	2 32 E	4
79	γ Coronæ Aust.	6, 6	18 59·7	37 12	1·400	143·3	1·93	10–8	300	P	4 18 E	3 53 E	3

Remarks.

In the column headed "Eyes" P denotes that the line joining the observer's eyes was parallel to that joining the components, and R that those lines were at right angles. The column headed "Hour-angles" gives the hour-angles between which the measures were made, and that headed "Weight" the value of each result on a scale of 1 to 5, 1 denoting the worst possible, and 5 the best possible, conditions. 2, companion dull blue. 8, observations in twilight. 11, 12, 13, measures difficult, owing to inequality of components; companion faint and bluish. 22, 23, 24, 35, 36, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 57, 69, 71, 72, 79, components noted equal. 26, 27, the preceding or south component evidently the brighter. 35, measures in full daylight. 36, measures in twilight. 37, the north component probably the brighter. 48, measures in sunlight and twilight. 55, 56, principal star white and companion blue. 73, 74, 75, principal star orange and companion pale blue.

The Peninsula, Windsor, N.S. Wales:
1901 *March* 25.

Errata.

Vol. LXI., page 342, *for* + 43° 33′ 39″·51, *read* + 43° 33′ 42″·51.
„ „ 344, Star 59, *for* 7·4506, *read* 7·4454.
„ „ 347, Star 155, *for* 3·9712. *read* 3·9612.